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Complete all of the problems in class. You may work in pairs and use the textbook.

Name and section: ____________________

Weekly Summary: Vectors and the Dot Product in 3 dimensions: (§11.1- §11.2)

- An equation of a sphere with centre (h, k, l) and radius r is:

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

- A three-dimensional vector is an ordered triple $a = (a_1, a_2, a_3)$ of real numbers. The a_i are called the components of a.
- Given points $P = (p_1, p_2, p_3)$ and $Q = (q_1, q_2, q_3)$, the vector from P to Q is given by $PQ = (q_1 - p_1, q_2 - p_2, q_3 - p_3)$.
- The length or norm of a vector $a = (a_1, a_2, a_3)$ is given by: $|a| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.
- Unit vectors: The following vectors are often used as building blocks for other vectors: $i = (1, 0, 0), j = (0, 1, 0), k = (0, 0, 1)$. For example, $(3, 5, 7) = 3i + 5j + 7k$.
- Addition of vectors: If $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ are vectors then $a + b$ is the vector defined by $a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$.
- Scalar Multiplication: If $a = (a_1, a_2, a_3)$ is a vector and c is a scalar (any real number), then the vector ca is defined by $ca = (ca_1, ca_2, ca_3)$.
- Let $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$. Then the *dot or scalar product* of a and b is defined as $a \cdot b = |a|\,|b| cos(\theta)$, where θ is the angle between a and b. The dot product is also given by $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$. NOTE: The dot product yields a number not a vector!
- Two vectors are orthogonal or perpendicular if $a \cdot b = 0$.
- Projection of b onto a: $proj_a b = \frac{a \cdot b}{|a|^2} a$
- Component of b onto a: $comp_a b = \frac{a \cdot b}{|a|}$

Workout Problems:

1. (§11.1.9) Determine whether the points $(1, 2, 3), (0, 3, 7), (3, 5, 11)$ are collinear.

2. (§11.1.15) Show that $x^2 + y^2 + z^2 + 2x + 8y - 4z = 28$ represents the equation of a sphere and find its radius.

3. (§11.2.8) Find $|a|, a + b, a - b, 3a + 4b$ if $a = (3, 2, -1), b = (0, 6, 7)$.

4. (§11.2.33) Find the values of x so that (x, 1, 2) and (3, 4, x) are orthogonal.

5. (§11.2.45) Find the projection of (1, 1, 1) onto (4, 2, 0).