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Complete all of the problems in class. You may work in pairs and use the textbook.

Name and section: ______________________________

Weekly Summary:

- The cross product: (§11.3)
  - Let $\mathbf{a} =< a_1, a_2, a_3 >$ and $\mathbf{b} =< b_1, b_2, b_3 >$. Then the *cross product* of $\mathbf{a}$ and $\mathbf{b}$ is defined as $\mathbf{a} \times \mathbf{b} = (|\mathbf{a}|\,|\mathbf{b}| sin(\theta))\,\mathbf{n}$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{n}$ is a unit vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ (with direction given by right hand rule).
  - Note: $\mathbf{a}\times\mathbf{b}$ is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. Also, $\mathbf{a}$ is parallel to $\mathbf{b}$ **iff** $\mathbf{a}\times\mathbf{b} = 0$. And $|\mathbf{a} \times \mathbf{b}|$ is equal to the area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.
  - Triple products: The *scalar triple product* of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is defined as

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) := \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

    Here, $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$ gives the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$. The *vector triple product* of $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is defined as $\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) := (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$.

- Equations of lines and planes (§11.4)
  - A *line* is defined as $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ (*vector equation*), where $\mathbf{r}_0$ is a point on the line and $\mathbf{v}$ is a direction vector of the line. If $\mathbf{r} =< x, y, z >$, $\mathbf{r}_0 =< x_0, y_0, z_0 >$, and $\mathbf{v} =< a, b, c >$, then the *parametric equations* are

$$\begin{aligned} x &= x_0 + ta \\ y &= y_0 + tb \\ z &= z_0 + tc \end{aligned}.$$

    The *symmetric equations* are given by

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \text{ if } a, b, c \neq 0.$$

  - A *plane* is described by its *linear equation* $ax + by + cz = d$, or $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$, where $\mathbf{n} =< a, b, c >$ is a vector orthogonal to the plane, $\mathbf{r}_0 =< x_0, y_0, z_0 >$ is a point in the plane, and $d = \mathbf{n} \cdot \mathbf{r} = ax_0 + by_o + cz_0$.

Workout Problems:

1. (§11.3) Find the cross product $a \times b$, where $a =< -2, 3, 4 >$ and $b =< 3, 0, 1 >$. (Problem 7)

2. (§11.3) Find the volume of the parallelepiped determined by the vectors a, b, and c, where $a =< 1, 0, 6 >$, $b =< 2, 3, -8 >$, and $c =< 8, -5, 6 >$. (Problem 19)

3. (§11.4) Find an equation of the plane passing through the points $(1, 0, -3), (0, -2, -4)$, and $(4, 0, 3)$. (Problem 29)