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Complete all of the problems in class. You may work in pairs and use the textbook.

Name and section: ______________________________

Weekly Summary:

- Power series: (§10.5)
  - $\sum_{n=0}^{\infty} c_n(x-a)^n$ is called a *power series* and define $a_n := c_n(x-a)^n$.
  - Ratio Test:
    The power series converges **iff** $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L|x-a| \overset{!}{<} 1$, i.e if $|x-a| < \frac{1}{L} := R$. Here, R is called the *radius of convergence*. Note that $|x-a| < R$ is equivalent to $-R+a < x < R+a$, i.e. $x \in (a-R, a+R)$. This is called the *interval of convergence*.
  - Note: Need to check the endpoints $x = a-R$ and $x = a+R$ explicitly!
- Representation of functions as power series: (§10.6)
  - Given $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ with radius of convergence R.
  - Then $f'(x) = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$, with radius of convergence R.
  - And $\int f(x)dx = \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1} + C$, with radius of convergence R.
  - Use differentiation and integration of f to get connection to geometric series and use $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ for $x < 1$.
- Taylor and Mclaurin series: (§10.7)
  - If f has the power series representation $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ for $|x-a| < R$, then $c_n = \frac{f^{(n)}(a)}{n!}$. The series is then called the *Taylor series* of f at a. $T_n(x) = \sum_{j=0}^{n} \frac{f^{(n)}(a)}{n!}(x-a)^n$ is called the *nth degree Taylor polynomial* of f at a.
  - If $a = 0$, we obtain the *Mclaurin series* of f, $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$.
  - Theorem: If $f(x) = T_n(x) + R_n(x)$ and $\lim_{n\to\infty} R_n(x) = 0$ for $|x-a| < R$, then $f(x) = T_\infty(x)$ on $|x-a| < R$.
  - Taylor's Inequality: If $|f^{(n-1)}(x)| \le M$ for $|x-a| < R$, then we have that
    $$|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1} \text{ for } |x-a| < R.$$
  - Use here: $\lim_{n\to\infty} \frac{x^n}{n!} = 0$.

Workout Problems:

1. (§10.5) Find the radius of convergence and the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{n+4}$.

2. (§10.6) Find a power series representation for the function $f(x) = ln(5-x)$ and determine the radius of convergence. (Problem 13)

3. (§10.7) Find the Taylor series for $f(x) = 1/x$ at $a = 1$. [Assume that f has a power series representation. Do not show that $R_n(x) \to 0$.] (Problem 9)