Group B - Proposition 4.5

<u>Proof</u>: (side to angle)

Given $AB > BC$, we need to prove that $\measuredangle ACB > \measuredangle A = \measuredangle CAB$

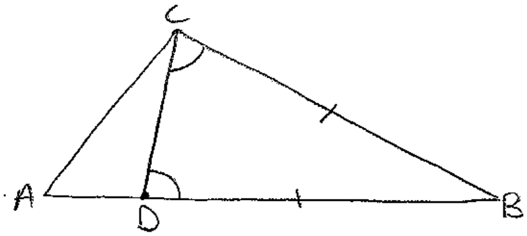
① Let $AB > BC$. Construct a point D such that $A * D * B$ and $BC \cong BD$ (by C-1).
Thus $\triangle BDC$ is an isosceles triangle by Prop 3.10

② By the exterior angle theorem (4.2), $\measuredangle BDC > \measuredangle ACD$ and $\measuredangle BDC > \measuredangle DAC$

③ Since $\triangle BDC$ is isosceles, $\measuredangle BDC \cong \measuredangle BCD$.
Thus $\measuredangle BCD > \measuredangle DAC$ by step 2.

④ By angle addition, $\measuredangle ACB = \measuredangle ACD + \measuredangle DCB$.

⑤ Hence, since $\measuredangle ACB > \measuredangle DCB$ and by step 3, we know that $\measuredangle ACB > \measuredangle CAD = \measuredangle CAB$.
Therefore, since $AB > BC \Rightarrow \measuredangle ACB > \measuredangle A = \measuredangle CAB$, the greater side lies opposite the greater angle.



<u>Proof</u>: (angle to side)

① Assume $\measuredangle C > \measuredangle A$

② There exists a point D on the line AB such that $DB \cong CB$.

③ Then exactly one of the following holds:

<u>Case 1</u>: $D * A * B$
Then $AB < DB$ and $DB \cong CB$, so $AB < CB$. Since $\measuredangle A$ is opposite CB, $\measuredangle A > \measuredangle C$, but this contradicts the assumption that $\measuredangle C > \measuredangle A$

<u>Case 2</u>: $D = A$
Then $AB \cong DB$, $DB \cong CB$, so $AB \cong CB$. Then $\triangle ABC$ is isosceles, so $\measuredangle C \cong \measuredangle A$, which is a contradiction.

<u>Case 3</u>: $A * D * B$

Then $AB > DB$ and $DB \cong CB$, so $AB > CB$. Since $AB > CB$ and $\measuredangle C > \measuredangle A$ and $\measuredangle C$ lies opposite AB, the greater angle lies opposite the greater side.