Group Zeta (Garcia, Hine, Maciejeski, Rohan)
MATH 467
Proposition 4.9 (Exercise 12)

Proposition 4.9: Hilbert's Euclidean parallel postulate $\Leftrightarrow$ if t is a transversal to l and m, $l \parallel m$, and $t \perp l$, then $t \perp m$.

PROOF:

$\Rightarrow$

1) Suppose first that Hilbert's postulate holds.
2) Let t be a transversal to lines l and m and assume $t \perp l$.
3) Let A be the point of intersection between l and t, and let B be the point of intersection between m and t (by Proposition 4.7).
4) Let C be a distinct point on l, so C≠A, and let D be any point on m on the opposite side of t from C.
5) $\angle$CAB is a right angle (by definition of perpendicular), so $\angle$ABD is also a right angle (by Proposition 4.8 and Proposition 3.15). Note: Proposition 4.8 proves that Hilbert's Euclidean parallel postulate is equivalent to the converse of the AIA Theorem (Theorem 4.1).
6) Therefore, $t \perp m$ (by definition of perpendicular).

$\Leftarrow$

1) Let l be a line and let B be a point not on l.
2) By Proposition 3.16, let t be a line perpendicular to l passing through B.
3) Consider two lines m and n through B and parallel to l. Therefore, by hypothesis, $t \perp n$ and $t \perp m$.
4) The perpendicular to t at B is unique (by Corollary to Theorem 4.1).
5) Therefore, n=m.