Jordan Rogers
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Homework 1 Revision

① According to Ahmes, the area of a circular disk is equal to the area of the square on $\frac{8}{9}$ths of the diameter. We know that the area of a square is s^2, where s is the length of a side of the square. So the area of the square is $s^2 = \left(\frac{8D}{9}\right)^2 = \left(\frac{8}{9}\right)^2 D^2$. Also, the area of a circle is $\pi r^2 = \pi \frac{D^2}{2^2}$. Using a circle of diameter 9cm, we get the area of the square to be $64 cm^2$ and the area of the circle to be $63.617 cm^2$.

If we compare the value of π to Ahmes' equation we get $\pi \frac{D^2}{2^2} = \text{Area}$ so $\pi = \text{Area}\left(\frac{2}{9}\right)^2 = 3.1605$. This value is close to the accepted value of π, 3.1416. We can also compare Ahmes' value, 3.1605, to other theories, like $22/7$ and $\sqrt{10}$. Ahmes' formula is less accurate than $22/7$ and it is more accurate than $\sqrt{10}$.

$$\frac{22}{7} = 3.14286$$
$$\text{Ahmes'} = 3.16049$$
$$\sqrt{10} = 3.16228$$

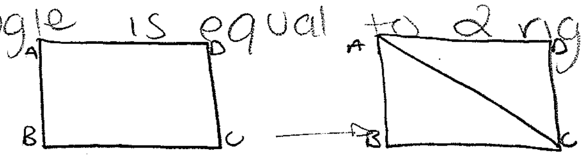
② If P and Q are any points on a circle with center O and radius OA, then $OP \cong OQ$ (pg. 44). There is a statement on pg. 17 that is similar to the statement on pg. 44. It states that "if P lies on the circle and $OP \cong OQ$, then Q also lies on the circle."

So, for a circle with center O to have a radius of OA, A must be a point on the circle. If we are given that other points, P and Q, also lie on the circle, we can say $OA \cong OP$ and $OA \cong OQ$. So $OP \cong OA \cong OQ$ and $OP \cong OQ$ because OP and OQ are both congruent to OA.

The statement on pg 17 is different from the statement on pg 44 because the pg 44 statement is the converse of the pg. 17 statement. On pg. 17 we are given that there is one point on the circle and that $OP \cong OQ$ (let us denote this as p). Then we must prove that Q also lies on the circle (let this be q). So this statement can be rewritten as $p \Rightarrow q$. The theorem on pg. 44 already states that there are 2 points, P and Q, on a circle with radius OA and we must prove that $OP \cong OQ$. So the statement on pg. 44 is $q \Rightarrow p$ which is the converse of the statement on pg. 17.

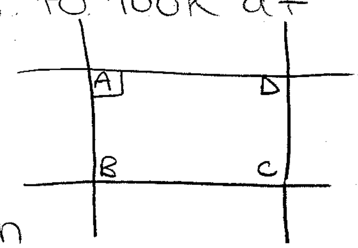
③ (i ⟹ ii) We are given a quadrilateral with four right angles. We know that a quadrilateral has four sides, so it has four angles. So if all four angles are right angles, then there is a quadrilateral with all angles congruent to one another.

(ii ⟹ i) We are given that all angles of a quadrilateral are congruent to each other. We can divide the quadrilateral into 2 congruent triangles because of the theorem that the sum of the angles in a triangle is equal to 2 right angles.



There are 2 triangles, so the angles add up to 4 right angles. All angles are congruent, so all angles must be right angles.

(i ⟹ iii) We are given that a quadrilateral has four right angles. To see if the quadrilateral is a parallelogram, we can use the theorem of transversals to look at the quadrilateral's angles.

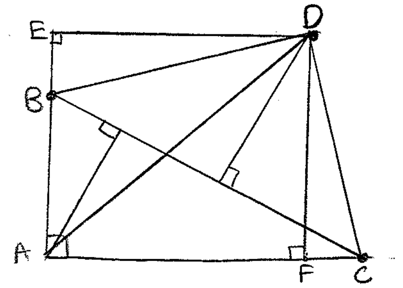


Adding ∡ABC and ∡DAB, with ∡DAB being a right angle, gives us a sum of 2 right angles. A sum of less than 2 right angles would mean that the line segments would eventually meet. We can determine that the quadrilateral is a parallelogram because the sum of the transversal's interior angles is eaual to 2 right angles. If the two angles summed to 2 right angles and the lines are parallel, then the last 2 angles must also be right angles.

(iii ⟹ i) We are given a parallelogram with at least one right angle. We know that opposite sides of a parallelogram are parallel and that opposite angles are congruent. ⟶

So the angle opposite the given right angle is also a right angle. We can then separate the parallelogram into 2 triangles. The sum of the angles in a triangle is equal to 2 right angles. There are 2 triangles, so the angles sum to 4 right angles. So the parallelogram has 4 right angles, and because a parallelogram is a quadrilateral, the quadrilateral has 4 right angles

(4)



In this case, F is between A and C and E is not between A and B. If ΔABC was an isosceles triangle then $\overline{AB} \cong \overline{AC}$, which would mean

$AB = AE - EB = AF - FC$; but $AC = AF + FE$,

or $AC = AF + FC = AE + EB$; but $AB = AE - EB$.

The only way $\overline{AB} \cong \overline{AC}$ would be if FC and EB were equal to zero - but this cannot happen because we are not in case 1. So $\overline{AB} \not\cong AC$, so the triangle is not isosceles.

The main point is that in a carefully drawn figure for Case 4, always E is outside the triangle and F inside, or vice versa.