Jordan Rogers
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Homework 2 Revision

1) Express in logical notation (quantifiers and connectives):
   a. n is an even number: $\exists x \in \mathbf{Z}[n = 2x]$
   b. For every number that is a perfect cube, there is a larger number that is even: $\forall x(\exists n[x = n^3] \Rightarrow \exists s \exists r[s = 2r \wedge s > x])$

2) Translate into standard mathematical English
   $\forall n \forall a \forall b[(n < a \wedge n < b) \Rightarrow n < ab]$: For all *n*, *a*, and *b*, if *n* is less than *a* and *n* is less than *b*, then *n* is less than the product *ab.*
   One could also say: For three numbers, if the first number is less than the second number and the first number is less than the third number, then the first number is less than the product of the second number and the third number.

3) Exercise 1, pg. 91:
   a. $\neg(P \vee Q) = \neg P \wedge \neg Q$
   b. $\neg(P \wedge \neg Q) = \neg P \vee Q$
   c. $P \Rightarrow Q$
      $\neg(\neg(P \Rightarrow Q))$ by logic rule 3
      $\neg(P \wedge \neg Q)$ by logic rule 4
      $\neg P \vee \neg\neg Q$ by logic rule 5
      $\neg P \vee Q$ by logic rule 3
      So $P \Rightarrow Q = \neg P \vee Q$

4) Exercise 2, pg 91: There exists a right angle that is not congruent to another right angle.

5) Exercise 4, pg 92:
   a. If a transversal *t* to lines *l* and *m* cuts out congruent alternate interior angles, then lines *l* and *m* are parallel.
   b. If lines *l* and *m* meet on one side of the transversal *t*, then the sum of the degree measures of the interior angles on that side of the transversal *t* is less than 180°.
6) Exercise 5, pg 92: There exist three distinct lines that are not concurrent.
   Proof.
   1. By axiom I-3, there exist 3 points that are not collinear.
   2. By axiom I-1, $\forall A \forall B((A \neq B) \Rightarrow \exists! m(A\mathrm{I}m \wedge B\mathrm{I}m))$;
      $\forall B \forall C((B \neq C) \Rightarrow \exists! n(B\mathrm{I}n \wedge C\mathrm{I}n))$;
      $\forall A \forall C((A \neq C) \Rightarrow \exists! s(A\mathrm{I}s \wedge C\mathrm{I}s))$
   3. Lines m, n, and s are distinct because if two were equal, the points would be collinear.
   4. By RAA hypothesis, assume that *m*, *n*, and *s* are concurrent lines. Then there exists a unique point that *m*, *n*, and *s* have in common, *P*.

5. By proposition 2.1 and by step 4, $P = A$, $P = B$, and $P = C$, so $A = B = C$. But this is a contradiction to the hypothesis that A, B, and C are not collinear (step 1).
6. Therefore, lines m, n, and s are not concurrent.

7) Exercise 6, pg 92:
   Proof (Proposition 2.4).
   By hypothesis we have some point *P*. We use the lemma that states that for every point *P* there exists two points that are not collinear with *P*. From this we know that there are two points, *A*, *B*, that are not collinear with *P*. Through these two points, *A* and *B*, there passes a unique line, by Euclid's First Postulate. Since *A* and *B* are not collinear with *P* we know that P is not incident to the line made by AB. So, for some points *P* there exist a line not passing through it.

   Proof (Proposition 2.5).
   By hypothesis we have some point *P*. We use the lemma that states that for every point *P* there exists two points that are not collinear with *P*. From this we know that there are two points, *M, N*, that are not collinear with *P*. Euclid's First Postulate states that given points, *P* and *M*, there exists a line $\overleftrightarrow{PM}$ passing through the points. Again, given points *P* and *N*, there exist a line $\overleftrightarrow{PN}$ that passes through them. By the lemma, we know that *P*, *M*, and *N* are not collinear, so $PM \neq PN$. Now, since $P \mathrm{I} \overleftrightarrow{PM}$ and $P \mathrm{I} \overleftrightarrow{PN}$ there exists two distinct lines through *P*.

8) Exercise 9, pg 92:
   a. Axiom 1 fails, but 2 and 3 hold, and it has the Euclidean parallel property
   b. Axioms 1, 2, and 3 hold, and it has the elliptic parallel property
   c. Axioms 1, 2, and 3 hold, and it has the hyperbolic parallel property
   d. Axioms 1, 2, and 3 hold, and it has the elliptic parallel property