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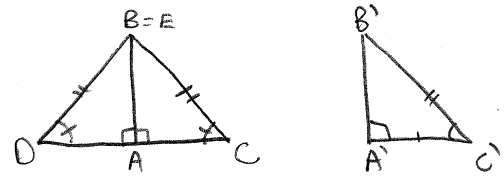
Homework 5 Revision

pg 193 (4) Prove Proposition 4.2



Proposition 4.2 states that two right triangles are congruent if the hypotenuse and a leg of one are congruent, respectively, to the hypotenuse and a leg of the other.

Proof:

(1) Let there be two right triangles $\triangle CAB$ and $\triangle C'A'B$ with $\angle A$ and $\angle A'$ being right angles and segment $BC \cong B'C'$. Also, let there be a segment $\overline{AD}$, on the opposite side of $\overrightarrow{AC}$ such that $AD \cong A'C'$, by C-1.

(2) So $\angle DAB$ is a right angle because $\angle DAB$ is supplementary to $\angle CAB$ and supplementary angles add up to $180°$ ($\angle CAB$ is a right angle).

(3) Let there be a point E on $\overrightarrow{AB}$ such that $AE \cong A'B'$

(4) Now we have $AD \cong A'C'$, $\angle DAB \cong \angle C'A'B$, and $AE \cong A'B'$, so we can apply C-6 (SAS) to say $\triangle EAD \cong \triangle B'A'C'$.

(5) Since $\triangle EAD \cong \triangle B'A'C'$ and we're given $BC \cong B'C'$, then $DB \cong BC$ and $\triangle DBC$ is an isosceles triangle

(6) So $\angle ADB \cong \angle ACB$ by proposition 3.10

(7) Now we have that $DB \cong BC$, $\angle ADB \cong \angle ACB$, and $\angle DAB \cong \angle CAB$. so by SAA (prop 4.1), $\triangle DBA \cong \triangle CBA$.

(8) Since $\triangle EAD \cong \triangle B'A'C'$ and $B=E$, then $\triangle BAC \cong \triangle B'A'C'$.

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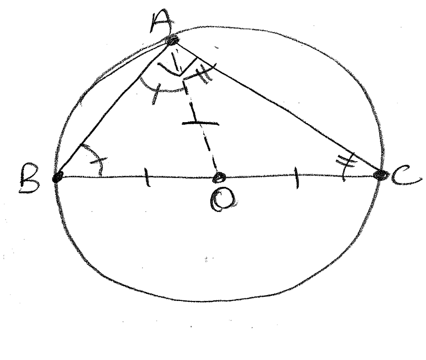
(19) Prove the theorem of Thales that in a Semi-Euclidean plane, an angle inscribed in a semicircle is a right angle.



Proof:

(1) Let O be the center of a circle and let there be points A, B, and C on the circle such that $B * O * C$.

(2) Then by definition of a radius, $OA \cong OB \cong OC$.

(3) we can construct two isosceles triangles, $\triangle AOB$ and $\triangle AOC$ such that $\sphericalangle ABO \cong \sphericalangle OAB$ and $\sphericalangle OAC \cong \sphericalangle ACO$, by prop. 3.10

(4) By prop 4.11 we know that every triangle has an angle sum $= 180°$, so $\sphericalangle BAC + \sphericalangle ACO + \sphericalangle OBA = 180°$

(5) so by angle addition, $\sphericalangle BAC = \sphericalangle BAO + \sphericalangle OAC$.

(6) By angle substitution, $\sphericalangle BAC + \sphericalangle ACO + \sphericalangle OBA = 180°$ becomes $\sphericalangle BAO + \sphericalangle OAC + \sphericalangle ACO + \sphericalangle OBA = 180°$

(7) we can say $2(\sphericalangle OAB) + 2(\sphericalangle OAC) = 180°$, so $\sphericalangle OAB + \sphericalangle OAC = 90°$, which is a right angle.

(8) Therefore, $\sphericalangle BAC = 90°$ and a right angle.

What happens if the plane is NOT semi-Euclidean?

Similary to the proof of the semi-Euclidean plane, and from part a of Saccheri's angle theorem, we can determine that $\sphericalangle BAO + \sphericalangle OAC + \sphericalangle ACO + \sphericalangle OBA \leq 180°$ Then we also know that $\sphericalangle OAB + \sphericalangle OAC \leq 90°$ and $\sphericalangle BAC \leq 90°$. Therefore, $\sphericalangle BAC$ would be acute and no longer a right triangle

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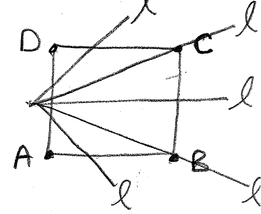
(30) State and prove a generalization of Pasch's Theorem for Saccheri and Lambert quadrilaterals (or, more generally, to convex quadrilaterals



Pasch's Theorem for Saccheri and Lambert quadrilaterals would be: If A, B, C, and D are distinct, noncollinear points that form a Saccheri/Lambert quadrilateral and l is any line intersecting AD in a point between A and D, then l also intersects either AB, BC, or CD. If B does not lie on l then l does not intersect both AB and BC. If D does not lie on l, then l does not intersect both BC and CD.

Proof:

(1) Either B lies on l or it does not. If it does then the theorem holds because B is a common point in both AB and BC, so l intersects both AB and BC.

- Either C lies on l or it does not. If it does, then the theorem holds because C is a common point in both BC and CD, so l intersects both BC and CD.
- Thus we may assume if both B and C lie on l, then that contradicts the fact that our quadrilaterals are convex. This is because if both B and C lie on l, and l intersects AD, that means that B and C are collinear.

(2) If B does not lie on l then B lies either on the same side of l as A or on the same side of l as D.

(3) If B lies on the same side of l as A, then B lies on the opposite side of l as D, so l intersects BC or CD and does not intersect AB.

4. If B lies on the same side of l as D, then B is on the opposite side of l as A. So l intersects AB and does not intersect BC.
5. So l does not intersect both AB and BC if B does not lie on l.
6. If C does not lie on l then C lies either on the same side of l as A or the same side of l as D.
7. If C lies on the same side of l as A, then C lies on the opposite side of l as D and l intersects CD and does not intersect BC.
8. If C lies on the same side of l as D, then C lies on the opposite side of l as A and l intersects BC or AB and does not intersect CD.
9. So l does not intersect both BC and CD if C does not lie on l.

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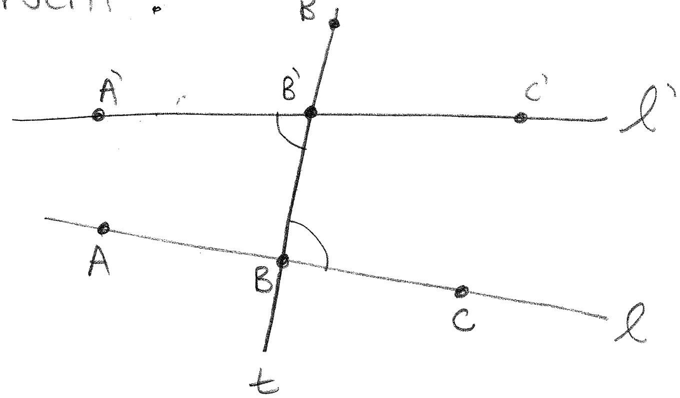
(32) Prove that corresponding angles are congruent if and only if alternate interior angles are congruent.



($\measuredangle A'B'B''$ and $\measuredangle ABB''$) and ($\measuredangle C'B'B''$ and $\measuredangle CBB''$) are corresponding angles

Proof: $\Longleftarrow$

1. Assume alternate interior angles are $\measuredangle A'B'B$ and $\measuredangle CBB''$
2. By alternate interior angle theorem 4.1, $\measuredangle A'B'B \cong \measuredangle CBB''$
3. Thus $\measuredangle A'B'B + \measuredangle A'B'B'' = 180°$ and $\measuredangle ABB'' + \measuredangle CBB'' = 180°$ by definition of supplementary angles
4. Now we can set $\measuredangle A'B'B + \measuredangle A'B'B'' = \measuredangle ABB'' + \measuredangle CBB''$
5. $\measuredangle A'B'B'' \cong \measuredangle ABB'$ by steps 2 and 4
6. Thus if alternate interior angles are congruent then corresponding angles are congruent

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$\Longrightarrow$

7. Now, by hypothesis we are given that corresponding angles $\measuredangle A'B'B'' \cong ABB''$
8. By definition of supplementary angles, $\measuredangle A'B'B + \measuredangle A'B'B'' = 180°$ and $\measuredangle CBB'' + \measuredangle ABB'' = 180°$

(9) Now we can set $\measuredangle A'B'B + \measuredangle A'B'B'' = \measuredangle CBB'' + \measuredangle ABB''$

(10) By step 7 and 9, $\measuredangle A'B'B \cong \measuredangle CBB''$

(11) Thus if corresponding angles are congruent then alternate interior angles are also congruent.