Math. 467 (Fulling) 6 March 2009

Midterm Test

Name: ______________________________

1. *(20 pts.)* Pronounce each of the following assertions true or false.

   (a) In Euclid's geometry, by definition, a line m is "parallel" to a line l if for any two points P, Q on m, the perpendicular distance from P to l is the same as the perpendicular distance from Q to l.

   (b) Euclid provided constructions for bisecting and trisecting any angle.

   (c) To "disprove" a statement means to prove the negation of that statement.

   (d) If line m is parallel to line l, then all points on m lie on the same side of l.

   (e) The notion of "congruence" for two triangles is an undefined term (in the Hilbert–Greenberg development of geometry).

2. *(14 pts.)* Prove that a triangle is isosceles **if and only if** two of its angles are equal.

3. *(28 pts.)*

   (a) State the three incidence axioms.

   (b) Consider a finite set S comprising N points. For each nonnegative integer N ($N = 0, 1, 2, 3, \ldots$), tell whether S is a model of the incidence axioms. If "Yes", tell which parallelism property the model satisfies.
   (Don't worry, this question does not have infinitely many parts. Eventually you will be able to say "For all $N \geq$ [some number], [something is true]." Proofs are not required, but brief explanations never hurt, especially if partial credit becomes an issue.)

4. *(10 pts.)* State **ONE** of these and illustrate with a diagram.

   (A) Pasch's Theorem

   (B) Crossbar Theorem

5. *(10 pts.)* Simplify $\sim \forall \mathrm{P} \exists l \, [(\mathrm{P} \; \mathtt{I} \; l) \vee \forall \mathrm{Q} \, (\mathrm{Q} \neq \mathrm{P} \;\Rightarrow\; \mathrm{Q} \; \mathtt{I} \; l)]$.
(Push the "$\sim$" in as far as you can! Truth or falsity of the statement is irrelevant.)

6. *(18 pts.)* Do **ONE** of these: *(no more than 10 points extra credit for doing both)*

(A) Define an *equivalence relation* and list 3 examples of equivalence relations that we have encountered in this course.

(B) State and prove one of the triangle congruence theorems, either ASA or SSS.
Warning: We do not yet have angle bisectors or segment bisectors (they come in Chapter 4), so you can't use them here.