1. Compute \( \lim_{t \to 2^-} \overrightarrow{r}(t) \), if \( \overrightarrow{r}(t) = \left( \frac{t^2 - 4}{t^2 - 5t + 6}, t^3 + 1 \right) \).

\[
\lim_{t \to 2^-} \overrightarrow{r}(t) = \left( \lim_{t \to 2^-} \frac{t^2 - 4}{t^2 - 5t + 6}, \lim_{t \to 2^-} t^3 + 1 \right) = \left( \lim_{t \to 2^-} \frac{(t - 2)(t + 2)}{(t - 2)(t - 3)}, 9 \right) = \left( -\frac{4}{9}, 9 \right)
\]

2. You are given the function \( f(x) \) and it is restricted to the interval \([0, 4]\). If \( f(0) = -0.5 \) and \( f(4) = 4.5 \), can you conclude that there is some value \( c \) such that \( 0 < c < 4 \) and \( f(c) = 2 \)? Justify your answer.

Nothing in this problem says that the function is continuous on the interval.

For example out function \( f(x) \) could be the following.

\[
f(x) = \begin{cases} 
-0.5 & \text{for } x = 0 \\
4.5 & \text{for } 0 < x \leq 4
\end{cases}
\]

If \( f(x) \) was continuous then, by the intermediate value theorem, we could conclude that there is some value \( c \) such that \( 0 < c < 4 \) and \( f(c) = 2 \).

3. Use the graph to answer the following questions.

(a) At what values of \( x \) is the function discontinuous? \( x = -2, x = 2, x = 4 \)

(b) Find the average rate of change from \( x = -1 \) to \( x = 3 \).

The average rate of change between two points is just the slope of the line (i.e. the slope of the secant line) connecting these points.

The points are \((-1, -1)\) and \((3, 1)\). So the slope is \( \frac{-1 - 1}{-1 - 3} = \frac{1}{2} \).

(c) Compute the slope of the secant line going thru the points at \( x = -1 \) to \( x = 3 \).

Same answer as found in part (b)
(d) At what values of x does the function not have a derivative? \( x = -2, x = 2, x = 4, x = 6 \)

(e) Compute the slope of the tangent line at \( x = 7 \).
Since the function is a straight line at \( x = 7 \), the slope of the tangent line is just the slope of line. Answer: \( m_{\text{tan}} = 5 \)

(f) What is the equation of the tangent line at \( x = -5 \)?
First you need the point, \((-5,3)\). Now you need the slope of the tangent line at \( x = -5 \).
I’ve drawn the line on the graph. Now just estimate the slope from the drawn line.

\[
\text{The } m_{\text{tan}} \approx 2.25 \\
\text{The equation of the tangent line is } y - 3 = 2.25(x + 5)
\]

(g) Compute \( f'(-1) \approx -2.2 \)

(h) For what values of x is the instantaneous rate of change equal to zero?
\( x = -3, x = 0 \)

4. Set up the difference quotient for \( f(x) = x^2 + 1 \) at \( x = 2 \). Now simplify the difference quotient.

Difference quotient: \[
\frac{f(2 + h) - f(2)}{h} = \frac{(2 + h)^2 + 1 - 5}{h}
\]

Now simplify: \[
\frac{(2 + h)^2 + 1 - 5}{h} = \frac{4 + 4h + h^2 + 1 - 5}{h} = \frac{4h + h^2}{h} = 4 + h
\]