1. The following is the graph of \( f'(x) \). The areas are given: \( A=7 \), \( B=12 \), and \( C=8 \). Also \( f(0) = 3 \). If there is not enough information to answer the question, then explain what additional information would be needed.

(a) \( f(2) = \)

By the Fundamental Theorem of Calculus, we know \[ \int_{0}^{2} f'(x)dx = f(2) - f(0). \] We also know the value of \( f(0) \), that was given above. All we need is the value of \[ \int_{0}^{2} f'(x)dx. \] This symbol means to do a Riemann sum from \( x=0 \) to \( x=2 \) and let the number of rectangles go to infinity.

Looking at the graph, we notice that this part of the graph is below the x-axis. This means all of the rectangles in this section will have “negative” height and hence will have “negative” area. Thus,

\[
\int_{0}^{2} f'(x)dx = f(2) - f(0).
\]

\[
-7 = f(2) - 3 \Rightarrow f(2) = 4
\]

(b) \( f(6) = \) Work this part in a similar fashion as part (a). It doesn’t matter if you start the integral at zero or at two.

\[
\int_{0}^{6} f'(x)dx = f(6) - f(0).
\]

\[
-7 + 12 = f(6) - 3 \Rightarrow f(6) = 8
\]

Just for fun we could compute that \( f(9) = 0 \)

(c) If the function \( f(x) \) is only defined on the interval \([0,9]\), find the absolute maximum and the absolute minimum of the function.

On the interval \([0,9]\), we know that the function \( f(x) \) is increasing between \([2,6]\) and decreasing everywhere else. Since \( f(x) \) is a continuous function on a closed and bounded interval, we know that it will have its absolute extrema at the critical values or at the end-points of the interval.

Using part (a) and (b) we can now conclude that the absolute maximum is 8 and the absolute minimum is -4

(d) If \( f'(x) \) is the velocity of an object, in ft/sec, and \( x \) is time measured in seconds, find the displacement of the object after 9 seconds.

The displacement is just \( \int_{0}^{9} f'(x)dx \).

Answer: \( \int_{0}^{9} f'(x)dx = -7 + 12 - 8 = -3 \) ft i.e the object went backwards 3 ft.
(e) If \( f'(x) \) is the velocity of an object and \( x \) is measured in seconds, find the total distance traveled by the object during the first 9 seconds.

The total distance traveled is to \( \int_{0}^{9} |f'(x)| \, dx \). i.e. we don’t want to have “negative” area.

Answer: \( \int_{0}^{9} |f'(x)| \, dx = | -7 | + 12 + | -8 | = 27 \, ft \)

2. Find the derivative of the following.

(a) \( g(x) = \int \frac{\sin(x)}{7} \cos(u^2 + 1) + 7u \, du = \)

\( g'(x) = \left[ \cos(\sin^2(x) + 1) + 7\sin(x) \right] \ast \cos(x) \)

(b) \( g(x) = \int_{x^3}^{2} \ln(u) \, du = \)

\( g'(x) = -\ln(x^3) \ast 3x^2 \)

\( \int_{0}^{x^2} 3te^t \, dt = \)

notice that the function \( y = 3te^t \) is always positive and it will go to infinity as \( x \) gets larger. This means that \( \int_{x^2}^{0} 3te^t \, dt \) will go to infinity as \( x \) goes to infinity. This is a L’Hopitals problem

\( \lim_{x \to \infty} \frac{\int_{x^2}^{0} 3te^t \, dt}{5xe^x} = \)

\( \lim_{x \to \infty} \frac{\int_{0}^{x^2} 3te^t \, dt}{5xe^x} = \lim_{x \to \infty} \frac{6x^3e^{x^2}}{(10x + 10x^3)e^{x^2}} = \frac{6}{10} = \frac{3}{5} \)

4. \( \int_{0}^{4} (3x + 4e^{4x}) \, dx = \left. \frac{3x^2}{2} + e^{4x} \right|_{0}^{4} = 24 + e^{16} - (0 + e^{0}) = 23 + e^{16} \)

5. \( \int_{1}^{6} (x - 2)(3x + 2) \, dx = \int_{1}^{6} 3x^2 - 4x - 4 \, dx = \left. x^3 - 2x^2 - 4x \right|_{1}^{6} = (6^3 - 2 \ast 6^2 - 24) - (1 - 2 - 4) = 125 \)

6. Find the area between the function \( f(x) = (x - 2)(3x + 2) \) from \( x = 1 \) to \( x = 6 \)

You have to break this integral into two parts. The function is below the x-axis between 1 and 2.

\( \text{Area} = \left| \int_{1}^{2} (x - 2)(3x + 2) \, dx \right| + \int_{2}^{6} (x - 2)(3x + 2) \, dx = \left| -3 \right| + 128 = 131 \)