Quiz #21  
MATH 151 Section _____  
Name: ________________________  

November 19, 2002

Be sure to show all work and make sure that you justify your answers. This is due at the start of class on Thursday.

1. Find two non-negative numbers $x$ and $y$ such that $x + y = 60$ for which the term $x^2y$ is maximized.

   $y = 60 - x$ and $J = x^2y$
   $J = x^2(60 - x) = 60x^2 - x^3$
   The domain of the function is $[0, 60]$
   $J' = 120x - 3x^2$
   $J'' = 120 - 6x$
   Solving for critical values
   $J' = 0$
   $0 = 3x(40 - x)$
   cv $x=0$ and $x= 40$
   $J''(0) > 0$ so this critical value is a min.
   $J''(40) < 0$ so this critical value is a max.

   Answer: $x=40$ and $y = 20$

2. A long rectangular sheet of metal, 12 in wide, is to be made into a rain gutter by turning up two sides so that they are perpendicular to the sheet. How many inches should be turned up in order to give the gutter its greatest capacity?

   The greatest volume will happen when the same amount of material is turned up on both sides. When we have the greatest volume, that is the same as saying the greatest area of a cross section.
   $A = x(12 - 2x) = 12x - 2x^2$
   The domain is $[0, 6]$
   $A' = 12 - 4x$ this means the cv is $x = 3$. $A'' = -4 < 0$ so whatever critical value we get will be a max. by the second derivative test.

   Answer: turn up 3 inches on each side.

3. A circular cylindrical container, open at the top, is to have a capacity of $24\pi$ in$^3$. The cost of the material used for the bottom of the container is 15 cents per square inch, and that of the material used for the side is 5 cents per square inch. If there is no waste of material, find the dimensions that will minimize the cost of constructing the can.

   $V = \pi r^2 h$
   $24\pi = \pi r^2 h$
   $h = \frac{24}{r^2}$
   $C = 15\pi r^2 + 5(2\pi r)h = 15\pi r^2 + 10\pi rh$
   $C = 15\pi r^2 + 10\pi r \frac{24}{r^2}$
   $C = 15\pi r^2 + \frac{240\pi}{r}$
   $C' = 30\pi r - \frac{240\pi}{r^2}$
   $C'' = 30\pi + \frac{480\pi}{r^3}$
   The second derivative will be positive for all positive values of $r$ (and some negative values also). We know that any realistic solution will have a positive value of $r$, so any positive critical values that we get will be minimum.

   Now solve $C' = 0$.
   $0 = \frac{30\pi r^3 - 240\pi}{r^2}$
   $0 = 30\pi r^3 - 240\pi$
   $240\pi = 30\pi r^3$
   $r^3 = 8$
   $r = 2$

   Answer: $r=2$ and $h = 6$