1. Find all values of $x$ which makes these vectors orthogonal (perpendicular). If there are no values possible, then explain why.

Two vectors are orthogonal provided their dot product is zero.

\[
<4x, -5> \cdot <x - 2, x + 7> = 4x(x - 2) - 5(x + 7) = 0
\]

\[
4x^2 - 8x - 5x - 35 = 0
\]

\[
4x^2 - 13x - 35 = (4x + 7)(x - 5) = 0
\]

Thus $x = -7/4$ or $x = 5$

2. If $\vec{m} = <3, 5>$, $\vec{n} = <10, 5>$ and $\vec{w} = <7, 2>$, find

(a) the vector projection of $\vec{w}$ onto $\vec{m}$.

\[
\text{proj}_m \vec{m} = \frac{\vec{m} \cdot \vec{n}}{|\vec{m}|^2} \vec{m} = \frac{21 + 10}{(\sqrt{9 + 25})^2} <3, 5 > = \frac{31}{34} <3, 5 > = \frac{93}{34}, \frac{155}{34}
\]

(b) the scalar projection of $\vec{m}$ onto $\vec{n}$.

\[
\text{comp}_n \vec{m} = \frac{\vec{m} \cdot \vec{n}}{|\vec{n}|} = \frac{30 + 25}{\sqrt{100 + 25}} = \frac{55}{\sqrt{125}} \approx 4.91934955
\]

3. Find a vector equation for the line passing through the points $A(1, 10)$ and $B(3, 16)$.

There are a lot of answers for this problem. First find a directional vector. I’m going to use the vector going from point A to point B: $\vec{n} = <2, 6>$. Now you need a point that is on the line. I’m going to use point A, but will express it as a vector: $\vec{x}_o = <1, 10>$. The vector equation is given by $\vec{r}(t) = \vec{x}_o + t\vec{n}$.

Answer: $\vec{r}(t) = <1, 10> + t <2, 6> = <1 + 2t, 10 + 6t>$

4. Does the point $(41, 103)$ lie on the line represented by the vector equation $\vec{r}(t) = <1 + 2t, 3 + 5t>$? justifying your answer.

If the given point is to be on the line, then there must be a value for $t$ such that

\[
\vec{r}(t) = <1 + 2t, 3 + 5t> = <41, 103>
\]

Now solving $1 + 2t = 41$ for $t$ gives $t = 20$. Since $3 + 5*20 = 103$, we know that the point is on the line.

5. Are these two lines parallel? Justify your answer. If they are not parallel, find the point where they intersect.

\[
L_1(t) = <3 + 2t, 4 - t> \quad L_2(s) = <1 + s, 2 - 5s>
\]

rewriting the lines in the form $\vec{r}(t) = \vec{x}_o + t\vec{n}$, gives:

\[
L_1(t) = <3, 4> + t <2, -1> \quad \text{and} \quad L_2(s) = <1, 2> + s <1, -5>
\]

Notice the directional vector for line 1, $<2, -1>$, is not a scalar multiple of the directional vector of line 2, $<1, -5>$. Thus the lines are not parallel.

To find the point where these lines intersect, i.e. $L_1(t) = L_2(s)$, just solve the system of equations: $3 + 2t = 1 + s$ and $4 - t = 2 - 5s$. Solving gives $s = -2/3$ and $t = -4/3$. Note: the values of $t$ and $s$ ARE NOT the point of intersection. To get the point of intersection, plug the value of $s$ into $L_2$ and the value of $t$ into $L_1$.

Answer: $(1/3, 16/3)$