Quiz #7

1. Find the derivatives of these functions. You do not have to simplify.

   (a)  \[ y = (x^3 - x)(x^2 + 7)^{10} \]
   \[ y' = (3x^2 - 1)(x^2 + 7)^{10} + (x^3 - x)10(x^2 + 7)^9(2x) \]

   (b)  \[ y = \frac{x^4 + 3}{x^3 - x} \]
   \[ y' = \frac{(x^3 - x)4x^3 - (x^4 + 3)(3x^2 - 1)}{(x^3 - x)^2} \]

2. Find the equation of the tangent line at \( x = 0 \) for \( y = \sqrt{x + 4} \)

   \[ y' = \frac{1}{2} (x + 4)^{-1/2} + 1 = \frac{1}{2\sqrt{x + 4}} \]

   To find the equation of the tangent line, you need a point and the slope of the tangent line. The point is found by plugging in the value of \( x \) into the function. Point \((0, 2)\). The slope is found by plugging in the value of \( x \) into the derivative. \( m_{\text{tan}} = \frac{1}{4} \)

   Answer: \( y - 2 = \frac{1}{4}(x - 0) \)

3. Find all values of \( x \), if any, where the function \( f(x) = (x^2 + 3)(x + 8) \) has an instantaneous rate of change of 3.

   This question says take the derivative and set it equal to 3 and solve for \( x \). You can make life a bit easier if you multiply out the function before taking the derivative. If you decide not to do this, then use the product rule to take the derivative and then simplify.

   \[ f(x) = x^3 + 8x^2 + 3x + 24 \]
   \[ f'(x) = 3x^2 + 16x + 3 \]

   Now
   \[ 3 = 3x^2 + 16x + 3 \]
   \[ 0 = 3x^2 + 16x \]
   \[ 0 = x(3x + 16) \]

   Thus \( x = 0 \) or \( x = -\frac{16}{3} \)