In [1]:
```
from sympy import *
from sympy.plotting import (plot, plot_parametric)
```

SLOPES AND TANGENTS OF PARAMETRIZED CURVES IN PYTHON

Recall that slopes of tangent lines can be found by the formula dy/dx = (dy/dt) / (dx/dt).

NOTE: The example here (and in future overviews) is NOT a copy/paste to solve the problems in lab. However, it will USE many of the features you will use to solve your problems, such as (in this case) differentiating vector functions, finding angles of tangent vectors, finding slopes and equations of tangent lines, and plotting.

EXAMPLE:

Given the curve parametrized by x=sin(t), y=sin(2t):

a) Plot the curve on the domain [0, 2pi].

b) Find the equation of the line tangent to the curve at t=5pi/6

c) Find the points on the curve where the tangent line is horizontal and where the tangent line is vertical.

d) Find the equations of the tangent lines at the origin, and plot both on the same graph.

Recall we now have a general strategy for solving Calculus problems in Python:

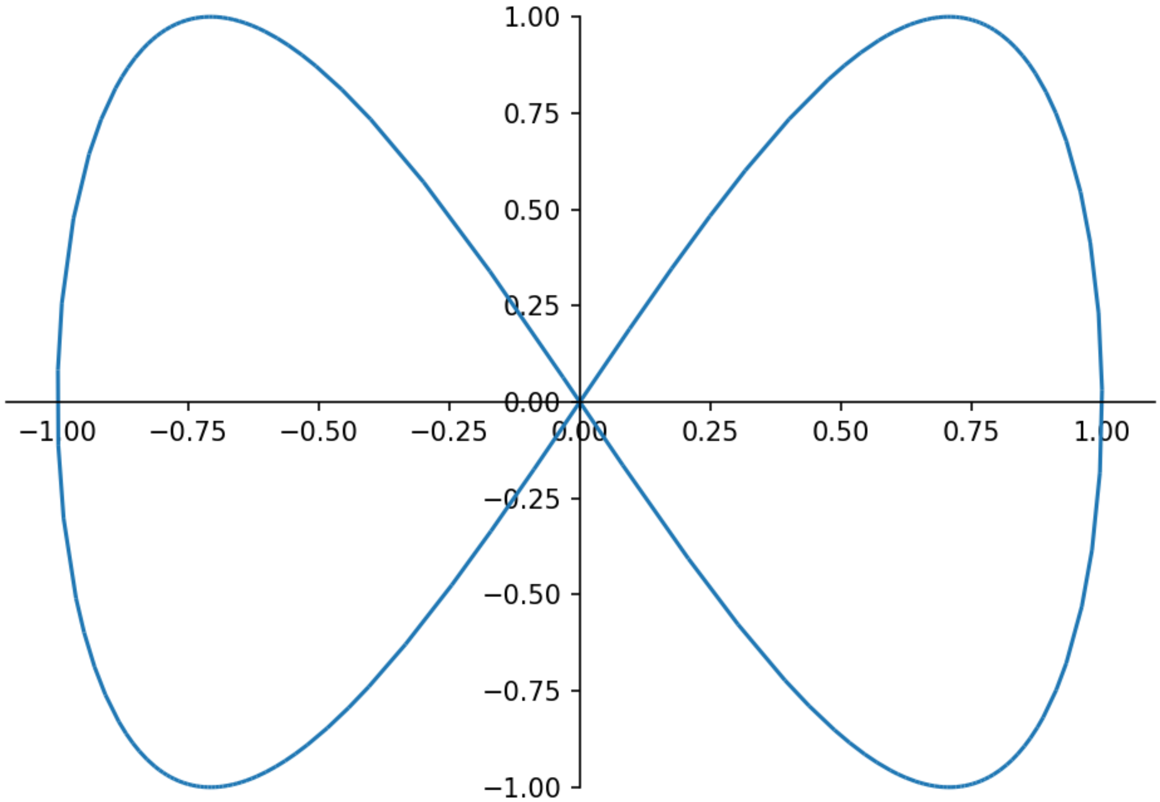
1) Write down the steps to solve them by hand first

2) List the Python command(s) needed to perform each step.

Part a just requires the plot_parametric function. As always, we start with the line below to tell Jupyter to put the graph in the notebook

In [2]:
```
matplotlib notebook
```

In [3]:
```
t=symbols('t',positive=True)
xoft=sin(t) # using xoft and yoft since we will need x for the tangent line equation in part b)
yoft=sin(2*t)
plot_parametric(xoft,yoft,(t,0,2*pi))
```



Out[3]: <sympy.plotting.plot.Plot at 0xa1b4410>

Steps to solve part b by hand (recall we need a slope and a point):

1. take the derivative using the formula stated at the beginning (**diff**)
2. Substitute t=5pi/6 into the derivative for the slope (**subs**)
3. Substitute t=5pi/6 into the original functions for the point (**subs**)
4. Print the equation y = y0 + m(x-x0) (**print**)

In [4]:
```
#Step 1
xp=diff(xoft,t)
yp=diff(yoft,t)
dydx=yp/xp
print('dy/dx=',dydx)
#Step 2
m=dydx.subs(t,5*pi/6)
print('The slope of the tangent line is',m,'or approximately',m.evalf())
#Step 3
x0=xoft.subs(t,5*pi/6)
y0=yoft.subs(t,5*pi/6)
print('The point on the tangent line is (',x0,',',y0,')')
#Step 4
x=symbols('x')
tanline=y0+m*(x-x0)
print('The equation of the tangent line is',tanline)
print('or approximately',tanline.evalf())
```

```
dy/dx= 2*cos(2*t)/cos(t)
The slope of the tangent line is -2*sqrt(3)/3 or approximately -1.154700538
37925
The point on the tangent line is ( 1/2 , -sqrt(3)/2 )
The equation of the tangent line is -2*sqrt(3)*(x - 1/2)/3 - sqrt(3)/2
or approximately -1.15470053837925*x - 0.288675134594813
```

Steps to solve part c by hand:

1. Find dy/dt and dx/dt (**diff**, though already done in the previous step)
2. For horizontal tangents, solve dy/dt=0 (**solve**) *-NOTE: the solve command does NOT find all solutions in (0,2pi). A short google search revealed a command* **solveset** *which will find all solutions on a specified* ***Interval****-see syntax below. Only needed for equations involving trig functions with period < 2pi*
3. For vertical tangents, solve dx/dt=0 (**solve**)
4. These only tell us WHEN the curve has horizontal and vertical tangents. To find WHERE, substitute the t-values into x and y (**subs**. ALSO note that solve returns a list of solutions, so we can use list comprehension-**for**-to find each set of points at one time!)

In [6]:
```
# Step 1 done in part b, called xp and yp
# Step 2
t_horiz=solveset(yp,t,domain=Interval(0,2*pi))  # Period of y' is < 2pi.  R
efer to the note above
print('Horizontal tangents when t=',t_horiz)
# Step 3
t_vert=solve(xp,t)  # period of x' is 2pi, so solve command finds all solut
ions
print('Vertical tangents when t=',t_vert)
# Step 4
horiz=[[xoft.subs(t,i),yoft.subs(t,i)] for i in t_horiz]
vert=[[xoft.subs(t,i),yoft.subs(t,i)] for i in t_vert]
print('Horizontal tangents at the points',horiz)
print('And vertical tangents at the points',vert)
```

```
Horizontal tangents when t= FiniteSet(pi/4, 3*pi/4, 5*pi/4, 7*pi/4)
Vertical tangents when t= [pi/2, 3*pi/2]
Horizontal tangents at the points [[sqrt(2)/2, 1], [sqrt(2)/2, -1], [-sqrt
(2)/2, 1], [-sqrt(2)/2, -1]]
And vertical tangents at the points [[1, 0], [-1, 0]]
```

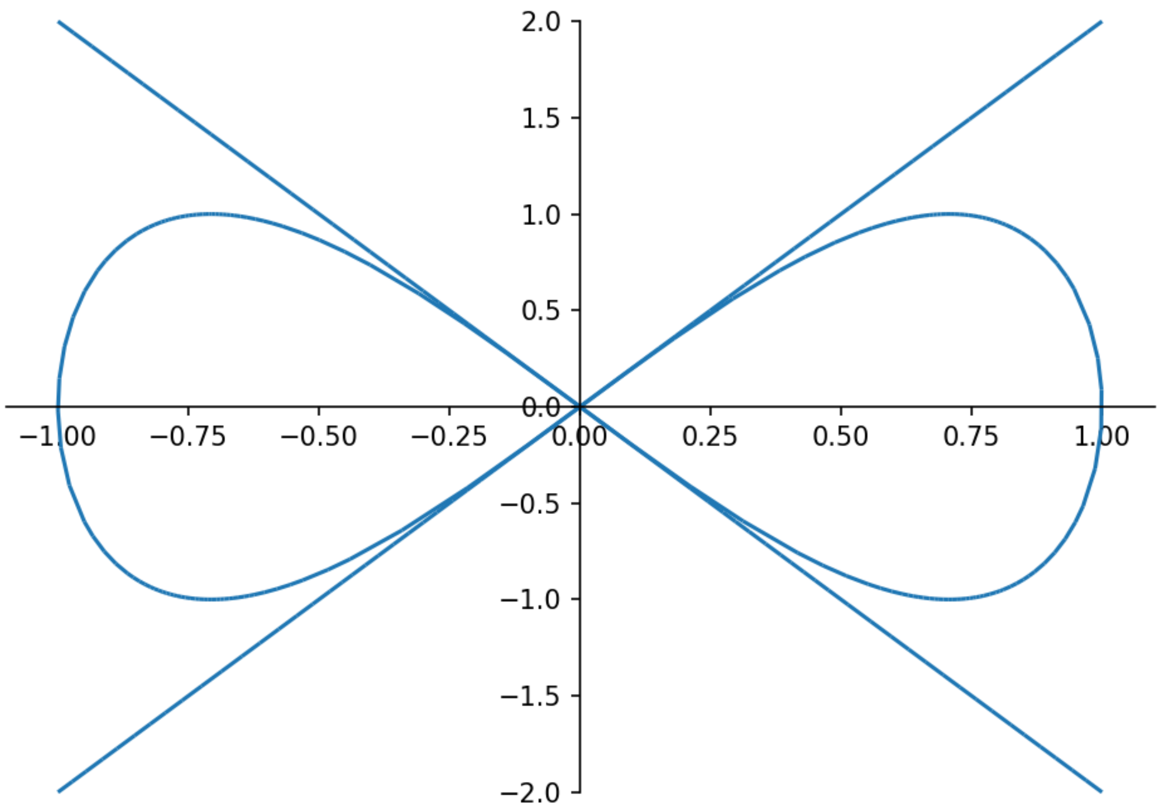
Steps to solve part d) by hand:

1. Start the same as part b (**diff** already done)
2. We don't have the t-values, just x=0 and y=0. Solve each for t (**solve**)
3. Once we have t, repeat steps 2 (**subs**) and 4 (**print**) from part b (NOTE that we already know x0=0 and y0=0)
4. Plot the parametrized curve (**plot_parametric**) and the tangent line equations (**plot**) in the same graph using the **extend** command

In [7]:
```
matplotlib notebook
```

In [10]:
```
x=symbols('x')
# Step 1 done in part b
# You may be able to tell the answer to step 2 by inspection.  That is fin
e.  If not...
tsoln=solve([xoft,yoft],t)
print(tsoln) #It should also be obvious that t=0 is a solution.  So our two
values are 0 and pi
# Step 3
m1=dydx.subs(t,0)
eq1=m1*x
m2=dydx.subs(t,pi)
eq2=m2*x
print('The equations of the tangent lines are y=',eq1,'and y=',eq2)
# Step 4
pcurve=plot_parametric(xoft,yoft,(t,0,2*pi),show=False)
ptangents=plot((eq1,(x,-1,1)),(eq2,(x,-1,1)),show=False) # Experiment to fi
nd appropriate domain
pcurve.extend(ptangents)
pcurve.show()
```

```
[(pi,)]
The equations of the tangent lines are y= 2*x and y= -2*x
```



In [ ]: