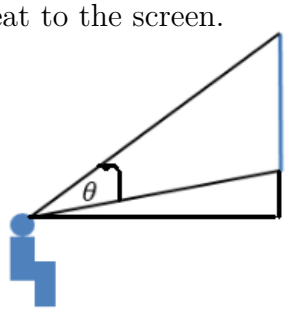
Math 151 Lab 5

Use Python to solve each problem.

1. Find the values of r for which $y = e^{rt}$ is a solution to the following differential equations
   a) $2y'' + 7y' + 6y = 0$
   b) $y'' - 4y' + 6y = 0$
   c) Note the values in part b) are complex. Substitute $y = e^{2x}(\cos(\sqrt{2}\,x) + \sin(\sqrt{2}\,x))$ into the left side of the differential equation and simplify to show it is a solution.
   d) Compare the numbers in this function to the real and imaginary part of the answers in part b). What do you notice?
2. Given the graph of the equation $x^3 + y^3 = 3xy$:
   a) Find $\dfrac{dy}{dx}$.
   b) Find the points (x and y coordinates) where the graph of the equation has a horizontal tangent line (**HINT:** force x and y to be real numbers in the **symbols** command).
   c) Use **plot_implicit** to plot the graph of the equation and the tangent lines from part b). Use the plot window $[-3, 3] \times [-3, 3]$.
3. An IMAX movie screen is 22 feet tall and is positioned with the bottom of the screen 4 feet above eye level (see figure below). Let x be the distance from your seat to the screen.



   a) Determine by hand an expression for θ in terms of x (HINT: inverse trig functions will be involved). Plot this expression in the domain $x \in [0.1, 30]$ to show there is a horizontal tangent line.
   b) Use calculus to find the x-value where the function has a horizontal tangent line. Give ONLY answers which make practical sense (exact and approximate).

   (Question 4 on the next page...)

4. Given the function $f(x) = (4x + 1)^{\cot x}$:

   a) Find the derivative directly in Python.

   b) Use the Python command **expand_log** to rewrite $g(x) = \ln(f(x))$, then compute $f(x) \cdot g'(x)$. Print both results.