Math 151 Lab 6

Use Python to solve each problem.

1. Given $f(x) = \begin{cases} x^2 + 5 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geq 1 \end{cases}$

   a) Plot f on the domain $x \in [0, 3]$ (NOTE: you do not have to plot any open/closed points, but mentally note where they are!)

   b) Use your graph to state (in a **print** statement) the absolute maximum and absolute minimum of f on $[0, 3]$ and the x-values at which they occur.

   c) Explain in a print statement why you cannot use the Extreme Value Theorem to answer part b).

2. Given $f(x) = \dfrac{3x-4}{x^2+1}$:

   a) Find the critical values of f (exact and approximate if necessary)

   b) Find the absolute maximum and absolute minimum of f on $[-2, 2]$.

   c) Plot the graph of the function on the interval above.

   d) Repeat parts b) and c) on the domain $[-2, 4]$.

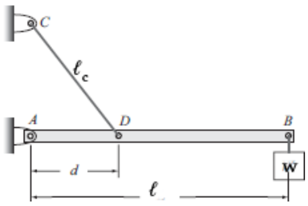
3. Let $f(x) = 100e^{-x/10} - 0.1x^2$.

   a) Find all (approximate) values c which satisfy the Mean Value Theorem on the interval $[0, 20]$.

   b) Illustrate the MVT by sketching the graph of f, the secant line from $x = 0$ to $x = 20$, and the tangent line at the c-value(s) found in a).
   (Problem 4 on the next page)

4. In the figure below, a beam ($\overline{AB}$) is attached to a wall at point A and to point C using a cable, and a weight w is hung at point B. Using principles of statics, the following equations can be used to find the tension T in the cable and the x and y components of the force at point A (A_x and A_y):

$$A_x - T\frac{d}{\ell_c} = 0$$
$$A_y - T\frac{\sqrt{\ell_c^2 - d^2}}{\ell_c} - w = 0$$
$$T\frac{\sqrt{\ell_c^2 - d^2}}{\ell_c}d - w\ell = 0$$



a) Write the forces T, A_x, and A_y each as functions of the constants d, ℓ_c, ℓ, and w.

b) Suppose $\ell = 10$ ft, $\ell_c = 5.5$ ft, and $w = 200$ lbs, so all forces in part a) are functions of d (where the cable attaches to the beam). You know that the force at A is $A = \sqrt{A_x^2 + A_y^2}$. Print $T(d)$ and $A(d)$ in a print statement and plot both on the domain $d \in [0.5, 5]$ (NOTE from the figure that $0 < d < \ell_c$)

c) Determine the minimum of $T(d)$ on the domain above, as well as where that minimum occurs (approximate).

d) Determine the minimum of $A(d)$ on the domain above, as well as where that minimum occurs (approximate)