

Math 152 Lab 5

Use Python to solve each problem.

1. Given the region R bounded by $y = (\cos x)^2$, $y = 0$, $x = 0$, and $x = \frac{\pi}{2}$:
 - (a) Graph the region R and find its area.
 - (b) Find the volume of the solid formed by rotating R about the x -axis.
 - (c) Find the volume of the solid formed by rotating R about the line $x = \frac{\pi}{2}$.
(NOTE: for all three parts, print the integrand, the indefinite integral, and the definite integral-exact and approximate where applicable)
2. Given $g(x) = \frac{x^3}{\sqrt{4-x^2}}$:
 - (a) Model “ u ”-substitution by substituting $x = \sqrt{4-w}$ (NOTE this is equivalent to $w = 4 - x^2$) into $\frac{g(x)}{-2x}$ (NOTE the denominator is w'), then integrate the resulting expression with respect to w , then re-substitute $w = 4 - x^2$.
 - (b) Model Integration by Parts by letting $u = x^2$ and $dv = \frac{x}{\sqrt{4-x^2}}$. Find du and v and then print the result from the IBP formula $(uv - \int v du)$.
 - (c) Model trig substitution by substituting $x = 2\sin(\theta)$ into the expression, multiply by dx , integrate the resulting expression* with respect to θ , then resubstitute $\theta = \arcsin\left(\frac{x}{2}\right)$.
*-NOTE: Python does not assume the square root to be positive, so print the resulting expression first and simplify by hand before integrating
 - (d) Your answers should not look the same. Simplify each of them to show that they are algebraically equivalent.

3. Given $f(x) = \frac{x^3 + 2}{x(x^2 + 4)^2}$:
- (a) Use the **apart** command to write the partial fraction decomposition of f , then integrate the partial fraction decomposition.
 - (b) Check your answer by integrating f directly.
4. Follow the strategy by hand for partial fractions on the expression $f(x) = \frac{x^3 - 1}{x^6 + x^3}$:
- (a) Factor the denominator (by hand or in Python) to determine the form of the partial fraction decomposition, then (by hand) clear the fractions and **simplify** the resulting expression in Python (it will also be useful to **collect** terms in powers of x)
 - (b) Create and solve a system of equations to find the coefficients, noting that the coefficients of x^3 should add to 1, the constants should add to -1 , and the coefficients of all other powers should add to 0.
 - (c) Check your answer by using the **apart** command directly.