Math 152 Lab 6

Use Python to solve each problem.

1. Use the **lambda** command to define $f(x) = \dfrac{\ln x}{1+x^2}$ as an inline FUNCTION. Most of the integrals in the parts below cannot be evaluated symbolically, so begin this problem by importing the **quad** command from the **scipy.integrate** package and using it:

   from scipy.integrate import quad

   (a) Find the area under the graph of f on the interval $[1, \infty)$.

   (b) Note f is negative on $(0, 1]$. Find the area above the graph of f on this interval.

   (c) Find the volume formed by rotating the region from part (a) about the x-axis

   (d) On your own, set up an integral to find the volume formed by rotating the region from part (a) about the y-axis. In a print statement, explain how you know this integral diverges (HINT: "To what shall I compare thee...")

2. The **average value** of a function f over an unbounded interval $[a, \infty)$ is given by

$$f_{avg} = \lim_{M\to\infty} \frac{1}{M-a} \int_a^M f(x)\,dx$$

   (a) If $f(x) = \arctan x$, compute $\int_0^\infty f(x)\,dx$ and find the average value of f over $[0, \infty)$.

   (b) If $f(x) = xe^{-x}$, compute $\int_0^\infty f(x)\,dx$ and find the average value of f over $[0, \infty)$. Explain why the average value make sense.

   (c) If $f(x) = \dfrac{1}{x}$, compute $\int_1^\infty f(x)\,dx$ and find the average value of f over $[1, \infty)$.

3. Given the sequence $a_n = \sqrt[n]{3^n + 5^n}$:

    (a) Find the first 10 terms of the sequence (approximate values) and guess the value of the limit based on these values.

    (b) Plot the first 40 terms of the sequence and guess the value of the limit based on the graph.

    (c) Find the limit of the sequence.

4. Find the first 40 terms of the sequence defined by

$$a_{n+1} = \begin{cases} \frac{1}{2}a_n & \text{if } a_n \text{ is even} \\ 3a_n + 1 & \text{if } a_n \text{ is odd} \end{cases}$$

    (a) If $a_1 = 11$

    (b) If $a_1 = 25$

    (c) Based on your answers to (a) and (b), predict what happens to the sequence if $a_1 =$ any positive integer.