Math 152 Lab 7

Use Python to solve each problem.

1. Given the series $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$:

   (a) Find $\lim_{n\to\infty} a_n$ (the sequence of the terms of the series).

   (b) Write the first 10 terms of the series and the first 10 partial sums of the series.

   (c) Plot the first 50 terms of the series and the first 50 partial sums on the same graph. (NOTE: use the **np.log** function for the logarithm, i.e., the numeric logarithm function instead of the symbolic logarithm function).

   (d) Based on your answers in (b), state a general formula for s_n, the nth partial sum, and find the sum of the series or show the series diverges.

   (e) Explain in a print statement how to show your answers for part (d) by hand (HINT: get a common denominator and use logarithm properties).

2. Suppose when a ball hits the ground with velocity v it rebounds with velocity $-kv$, where $0 < k < 1$.

   (a) If the ball is dropped from a height H, what is the total distance the ball travels if allowed to bounce until it (practically) comes to a complete stop?

   (b) For how much time does the ball bounce?
   (**Key formulas**: $v_f^2 - v_0^2 = 2gs$, $s = \frac{1}{2}gt^2$. Use a symbolic g for this part. ALSO remember that after the ball hits the floor the first time, what goes up also comes down!)

   (c) Using your answers to (a) and (b), find the total distance and time if $k = 0.95$ and $H = 2$ meters (use $g = 9.8$).

3. Given the series $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$:

   (a) Compute an appropriate integral to test the convergence of this series.

   (b) If Python was able to compute the integral, use the result to state whether the series converges or not. If Python was not able to compute the integral, state an appropriate series with which to use the Comparison or Limit Comparison Test. (You do not have to show this test unless you want to check your answer is correct!)

   (c) Repeat parts (a) and (b) for $\sum_{n=1}^{\infty} \frac{e^n+1}{ne^n+10}$.
   (Problem 4 on the next page...)

4. Given $a_n = \dfrac{n}{4^n}$

(a) Using the Remainder Estimate for the Integral Test to determine N, the minimum number of terms needed to sum $\displaystyle\sum_{n=1}^{\infty} a_n$ to within .00005.

(b) Use s_N to estimate the sum of the series to within .00005.

(c) Confirm your answer by finding the exact sum of the series s and computing $s - s_N$.