Math 152 Lab 8

Use Python to solve each problem.

1. Given $a_n = \dfrac{1}{(2n+1)^3}$

   (a) Plot a_n in the window $y \in [0, .000001]$ to determine the fewest number of terms needed to sum $\sum_{n=0}^{\infty}(-1)^n a_n$ to within .000001 (based on the Alternating Series Estimation Theorem).

   (b) Use **solve** to confirm your graphical answer in part (a)

   (c) Find the sum of the series to within .000001.

   (d) Let S equal your value from (c). Compute $(32S)^{1/3}$ and explain the significance of your answer.

2. Given the power series $\sum_{n=0}^{\infty}(-1)^n \dfrac{x^{6n+5}}{2n+1}$:

   (a) Simplify $\left|\dfrac{a_{n+1}}{a_n}\right|$ and find the limit.

   (b) State the values of x for which this limit is less than 1 (you may do this by hand or by using **reduce_inequalities** in Python). Your answer will be an interval which is cenetered at $x = 0$; how far from 0 is each endpoint? (this is called the *radius of convergence* of the series). Substitute each endpoint into the series to determine whether the series converges or not.

   (c) It can be shown that for values of x where the series converges, it converges to $f(x) = x^2 \arctan(x^3)$ (we call these values of x the *interval of convergence* of the series). To illustrate this, find s_1, s_3, and s_5. Plot these three polynomials and f on the same set of axes in its interval of convergence (use default range).

3. Given the power series $\sum_{n=0}^{\infty}(-1)^n \left(1 + \dfrac{1}{2^{n+1}}\right) x^n$:

   (a) Simplify $\left|\dfrac{a_{n+1}}{a_n}\right|$ and find the limit.

   (b) State the radius of convergence and the endpoints. Substitute to show whether each endpoint is in the interval of convergence or not.

   (c) It can be shown that the series converges to $f(x) = \dfrac{2x+3}{x^2+3x+2}$ on its interval of convergence. To illustrate this, find s_5, s_{10}, and s_{15}. Plot these three polynomials and f on the same set of axes in its interval of convergence ($y \in [-5, 5]$).

(d) Find the partial fraction decomposition of f. Based on this, explain in a print statement how to obtain the power series by hand (in general-you do not have to list the steps).