

Name _____
Section _____

Math 151
Common Exam # 3, Version A
Spring 1998

No calculators are allowed on this exam. The SCANTRONS will be collected at the end of the first hour. Put your name, Section number, and the version of your exam on your SCANTRON.

Part I.

Each problem is worth 4 points. Mark your answers on your SCANTRON form. Put your name and Section number on your SCANTRON. Indicate on your SCANTRON form whether you have Version A or B of the exam. For your own record, also mark your answers on this exam. The exam will be returned to you; the SCANTRON will not.

1. If $f(x) = \tan^{-1}(x^2)$, then $f'(x) =$

a) $\frac{2x}{1+x^2}$ b) $\frac{x^2}{1+x^4}$ c) $\frac{2x}{1+x^4}$ d) $\frac{1}{1+x^4}$

2. $\sin(\tan^{-1} \frac{3}{4}) =$

a) $\frac{3}{5}$ b) $\frac{4}{5}$ c) $\frac{1}{2}$ d) $\frac{3}{4}$

3. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, what is the smallest possible value for $f(4)$?

a) 12 b) 20 c) 32 d) 16

4. $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} =$

- a) $-\frac{1}{6}$ b) 0 c) $\frac{1}{6}$ d) 1

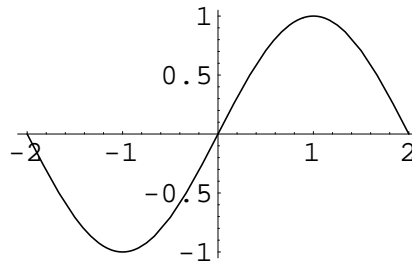
5. On which of the following intervals is $f(x) = x^2(1 - x)$ increasing?

- a) $(-\frac{2}{3}, \frac{2}{3})$ b) $(0, 1)$ c) $(-1, 0)$ d) $(0, \frac{2}{3})$

6. The absolute minimum value of $f(x) = x^3 - 12x$ on the interval $[-3, 5]$ is

- a) 0 b) -16 c) -14 d) -18

Below is the graph of the **derivative** of a function $f(x)$. Use the graph to answer the next two questions.



7. At which of the following points does $f(x)$ have a point of inflection?

- a) -2 b) 0 c) 1 d) 2

8. At which of the following points does $f(x)$ have a local maximum?

- a) -2 b) -1 c) 0 d) 1

9. What is $\sum_{i=3}^6 (2i - 1)$?

- a) 24 b) 27 c) 39 d) 32

10. What is $f(x)$ if $f'(x) = 12x^2 - 24x + 1$ and $f(1) = -2$?

- a) $4x^3 - 12x^2 + x + 5$ b) $4x^3 - 12x^2 + 6$ c) $12x^3 - 24x^2 + x + 9$ d) $4x^3 - 12x^2 + x - 5$

11. $\int_{-1}^0 (x + 1)^3 dx =$

- a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) 0 d) $-\frac{1}{4}$

12. If $h(x) = \int_1^{\sqrt{x}} \frac{s^2}{s^2 + 1} ds$, what is $h'(x)$?

- a) $\frac{x}{1+x}$ b) $\frac{x^2}{1+x^2}$ c) $\frac{\sqrt{x}}{2(1+x)}$ d) $\frac{x\sqrt{x}}{2(1+x^2)}$

13. The antiderivative of $\frac{x^2 + 1}{\sqrt{x}}$ is

- a) $\frac{x^2 + 3}{3\sqrt{x}} + c$ b) $\frac{2}{5}x^2\sqrt{x} + 2\sqrt{x} + c$ c) $\frac{2}{5}x^2\sqrt{x} - \frac{2}{3x\sqrt{x}} + c$ d) $\frac{2}{5}x^2\sqrt{x} - 2\sqrt{x} + c$

Part 2.

Show all your work.

14. Find the absolute maximum and minimum values of $f(x) = \sin x + \cos x$ on the interval $[0, \pi/3]$.

15. Find all intervals where $f(x) = 2x^2 - x^4$ is increasing, all intervals where it is decreasing, all intervals where it is concave upward, all intervals where it is concave downward, all local maximums and minimums and all points of inflection.

16. Polonium-210 has a half-life of 140 days.

a) If a sample has a mass of 200 mg, find a formula for the mass that remains after t days.

b) Find the mass after 100 days.

c) When will the mass be reduced to 10 mg?

17. Find $\lim_{x \rightarrow 0^+} (1 - 2x)^{1/x}$.

18. A box with an open top is to be constructed from a square piece of cardboard, 3 ft wide, by cutting squares from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

19. A fence 8 ft tall runs parallel to a tall building at a distance of 8 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?