MATH 151, FALL SEMESTER 2004
COMMON EXAMINATION II - VERSION B

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–13), mark the correct choice on your ScanTron form using a No:2 pencil. *For your own record, mark your choices on the exam itself*. ScanTrons will be collected from all examinees after one hour, and will *not* be returned. *No student will be allowed to turn in his/her scantron before the end of the first hour*.

2. Calculators may *not* be used in Part 1. The use of calculators is permitted *only after the first hour has elapsed and all ScanTrons have been collected*.

3. In Part 2 (Problems 14–18), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

QN	PTS
1–13	
14	
15	
16	
17	
18	
TOTAL	

Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points**. *Calcluators are* **not** *allowed for this part of the exam.*

1. Evaluate $\lim_{x\to\infty} 3^{-x}$.

(a) ∞

(b) 1

(c) -1

(d) 0

(e) $-\infty$

2. Determine the domain of the function $f(x) = \ln(|x|)$.

(a) $x > 0$

(b) $x > 1$

(c) $x \neq 0$

(d) $x < 0$

(e) all real numbers

3. If $\log_3(3x + 2) = 2$, what is x?

(a) 0

(b) 4/3

(c) 3/2

(d) 7/3

(e) 5/2

4. Given that $f(x) = \sqrt{1 - x}$, find the range of f^{-1} (the inverse of f).

(a) $(-\infty, \infty)$

(b) $[1, \infty)$

(c) $(-\infty, 1]$

(d) $[-1, 1]$

(e) $[0, \infty)$

5. Suppose that $f(x) = x^5 + x^3$ and let g denote its inverse. What is $g'(2)$? (Note: $f(1) = 2$ and $f(2) = 40$.)

(a) $1/92$

(b) $1/8$

(c) $-92/1600$

(d) $1/(5(40)^4 + 3(40)^2)$

(e) cannot be determined with the information given

6. Differentiate the function $f(x) = x^2 \tan x$ with respect to x.

(a) $2x \sec^2 x$

(b) $2x \tan x + x^2 \sec x \tan x$

(c) $2x \tan x + x^2 \sec^2 x$

(d) $2x \tan x - x^2 \sec^2 x$

(e) $2x \tan x - x^2 \sec x \tan x$

7. Differentiate the function $f(x) = 2\sqrt{e^x}$ with respect to x.

(a) $2\sqrt{e^x}$

(b) $\sqrt{e^x}$

(c) $e^{\sqrt{x}}$

(d) $\sqrt{e^x}/2$

(e) $2e^{\sqrt{x}}$

8. Let $\mathbf{r}(t) = \langle \cos^2 t, t \rangle$ be a vector function representing the position of a particle at time t. Find the acceleration of the particle at $t = \pi/3$.

(a) $\langle -1, 1 \rangle$

(b) $\langle 1, 1 \rangle$

(c) $\langle -1, 0 \rangle$

(d) $\langle 1, 0 \rangle$

(e) $\langle -\sqrt{3}, 0 \rangle$

9. Find $D^{28}(\cos x)$ (*i.e.*, the twenty-eighth derivative of $\cos x$).

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

(e) $\sin x \cos x$

10. Find the linear (*i.e.,* tangent-line) approximation $L(x)$ to the function $f(x) = \sin(x^2)$ at $a = \sqrt{\pi}$.

(a) $L(x) = \sqrt{\pi} - x$

(b) $L(x) = -2\sqrt{\pi}\, x$

(c) $L(x) = -2\pi + 2\sqrt{\pi}\, x$

(d) $L(x) = 2\pi - 2\sqrt{\pi}x$

(e) $L(x) = 2\sqrt{\pi} - 2x$

11. Suppose that $x > 0$ and let s denote the distance between the points $(x, 0)$ and $(0, 1)$. If x is changing with time and $\dfrac{dx}{dt} = 2$, then $\dfrac{ds}{dt} = ?$

(a) $2x$

(b) 2

(c) $\dfrac{x}{(x^2+1)}$

(d) $\dfrac{2x}{\sqrt{x^2+1}}$

(e) $\dfrac{4x^2}{x^2+1}$

12. $2^{-x} + 2^{-x} = ?$

(a) 2^{-2x}

(b) 4^{-2x}

(c) 2^{x^2}

(d) 2^{1-x}

(e) 4^{-x}

13. Let f be a differentiable function, and let C denote the graph of $y = f(x)$. It is known that C passes through the point $P(2, 2)$, and that the slope of the tangent to C at P is a number between 3 and 5. If one begins Newton's method (to solve the equation $f(x) = 0$) with $x_1 = 2$, which of the following intervals will contain the next iterate x_2?

(a) $(-1, 0)$

(b) $(0, 1)$

(c) $(1, 2)$

(d) $(2, 3)$

(e) $(3, 4)$

Part 2 (54 points)

The use of a calculator is permitted for this part of the exam, but all work must be shown in order to receive credit. Refer to the front page for further instructions.

14. (12 points) Let C be the curve given by the equation

$$(x^2 + y^2)^2 = 2(x^2 - y^2) + 6x + 7.$$

Use implicit differentiation to find the slope of the tangent to C at the point $(2, 1)$.
Present your work methodically and clearly; merely relaying an answer reported by your calculator is unacceptable.

15. (14 points) Consider the curve C given by the parametric equations

$$x(t) = te^{-t}, \qquad y(t) = (t+1)^{1/3}, \quad -\infty < t < \infty.$$

(i) (8 points) Find $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$.

(ii) (3 points) Find $\dfrac{dy}{dx}$ <u>in terms of the parameter t</u>.

(iii) (3 points) Obtain an equation of the tangent line to C at the point $(0, 1)$.

16. (12 points) Suppose that a and b are constants, neither of which is zero. Compute the first and second derivatives of the function $f(x) = e^{bx}\sin(ax)$. Show all your steps clearly.

17. (`10 points`) A television camera is positioned 4000 feet from the base of a rocket launching pad. A rocket rises vertically and its speed is 600 feet per second when it has risen 3000 feet. Assuming that the camera is always kept focused on the rocket, how fast is the camera's angle of elevation changing at that same moment? Include units with your answer.

18. (`6 points`) The linear (*i.e.,* tangent-line) approximation to a function f at $a = 1$ is given by $L(x) = 3x + 2$. Find the linear approximation to the function $H = \sqrt{f}$ at $a = 1$. Explain your reasoning clearly and carefully.