Fall 2005 Math 151
Exam 2B: Solutions
Mon, 31/Oct ©2005, Art Belmonte

1. (d) Now $f'(x) = 3e^{3x} + 4e^{2x} - 3e^{x} + 2\cos x - 2x\sin x$, whence $f'(0) = 3 + 4 - 3 + 2 - 0 = 6$.

2. (c) The product rule gives
$$\frac{d}{dx}\left(x^2 \sin 4x\right) = (2x)(\sin 4x) + \left(x^2\right)(4\cos 4x).$$

3. (b) Use the fact that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$.
$$\begin{aligned}\lim_{x\to 0} \frac{x(1-2\cos 3x)}{\sin 3x} &= \lim_{x\to 0} \frac{\frac{x}{3x}(1-2\cos 3x)}{\frac{\sin 3x}{3x}} \\ &= \lim_{x\to 0} \frac{\frac{1}{3}(1-2\cos 3x)}{\frac{\sin 3x}{3x}} \\ &= \frac{\frac{1}{3}(1-2)}{1} = -\frac{1}{3}\end{aligned}$$

4. (d) We have
$$\begin{aligned} F'(x) &= f'(g(x))\, g'(x) \\ F'(2) &= f'(g(2))\, g'(2) \\ &= f'(5)\, g'(2) \\ &= (11)(4) = 44. \end{aligned}$$

5. (e) We have
$$\begin{aligned}\frac{d}{dt}\left(xy^3\right) &= \left(\frac{dx}{dt}\right)\left(y^3\right) + (x)\left(3y^2\frac{dy}{dt}\right) \\ &= (2)(8) + (3)(3)(4)(-4) \\ &= 16 - 144 = -128.\end{aligned}$$

6. (c) Solve $y = f(x)$ for x.
$$\begin{aligned} y = f(x) &= \frac{3x+2}{2x+7} \\ 2yx + 7y &= 3x + 2 \\ (2y-3)x &= 2 - 7y \\ x &= \frac{2-7y}{2y-3} = \frac{7y-2}{3-2y}\end{aligned}$$
Thus $f^{-1}(x) = \frac{7x-2}{3-2x}$.

7. (b) When $t = 2$, we have
$$\begin{aligned} h'(t) &= 3\left(t^3 - t^2 - 2t + 1\right)^2\left(3t^2 - 2t - 2\right) \\ &= 3(8-4-4+1)^2(12-4-2) \\ &= 3(1)(6) = 18.\end{aligned}$$

8. (d) The limiting expression is a decaying exponential. Therefore, as $x \to -\infty$, we see that $e^{x-1} \to 0$.

9. (e) Use log properties. Be sure to check your answer.
$$\begin{aligned}\log_3 x + \log_3\left(x^3\right) &= 8 \\ 4\log_3 x &= 8 \\ \log_3 x &= 2 \\ x &= 3^2 = 9\end{aligned}$$
This checks out via substitution into the original equation.

10. (c) By inspection, $f(1) = 7$. Thus $g(7) = 1$. Accordingly,
$$\begin{aligned} g'(7) &= \frac{1}{f'(g(7))} \\ &= \frac{1}{f'(1)} \\ &= \frac{1}{\left(3x^2+1\right)\big|_{x=1}} = \frac{1}{4}.\end{aligned}$$

11. (b) Given position $\mathbf{r}(t) = \left[t^3 - 4t^2 + 2, 2t^2 - 3t\right]$, we have $\mathbf{r}(3) = [27 - 36 + 2, 18 - 9] = [-7, 9]$.

12. (e) Velocity, the derivative of position, is a vector.
$$\mathbf{v}(t) = \mathbf{r}'(t) = \left[3t^2 - 8t, 4t - 3\right]$$
Thus $\mathbf{v}(3) = [27 - 24, 12 - 3] = [3, 9]$. The speed is the *magnitude* of velocity: $\sqrt{9+81} = \sqrt{90}$ or $3\sqrt{10}$, a scalar.

13. (e) The acceleration is $\mathbf{a}(t) = \mathbf{v}'(t) = [6t - 8, 4]$. Therefore, $\mathbf{a}(3) = [18 - 8, 4] = [10, 4]$.

14. Use product and chain rules, as applicable.

    (a) Given $f(x) = \frac{\tan 2x}{\sin 3x + \cos 4x}$, we have
$$f'(x) = \frac{(\sin 3x + \cos 4x)\left(2\sec^2 2x\right) - (\tan 2x)(3\cos 3x - 4\sin 4x)}{(\sin 3x + \cos 4x)^2}.$$

    (b) Given $f(x) = \sqrt{1 + e^{x^2+3x}} = \left(1 + e^{x^2+3x}\right)^{1/2}$, we have
$$f'(x) = \frac{1}{2}\left(1 + e^{x^2+3x}\right)^{-1/2} e^{x^2+3x}(2x+3)$$
or $f'(x) = \frac{(2x+3)e^{x^2+3x}}{2\sqrt{1+e^{x^2+3x}}}$.

15. Implicitly differentiate $2\left(x^2+y^2\right)^2 = 25xy$ with respect to x, then substitute the data $(x, y) = (2, 1)$.
$$\begin{aligned} 4\left(x^2+y^2\right)\left(2x + 2y\frac{dy}{dx}\right) &= 25y + 25x\frac{dy}{dx} \\ \text{Substitute data: } 4(4+1)\left(4 + 2\frac{dy}{dx}\right) &= 25 + 50\frac{dy}{dx} \\ 80 - 25 &= 10\frac{dy}{dx} \\ \text{At stated point: } y' &= \tfrac{55}{10} = \tfrac{11}{2}\end{aligned}$$
The point-slope formula gives $y - 1 = \frac{11}{2}(x - 2)$ or $y = \frac{11}{2}x - 10$.

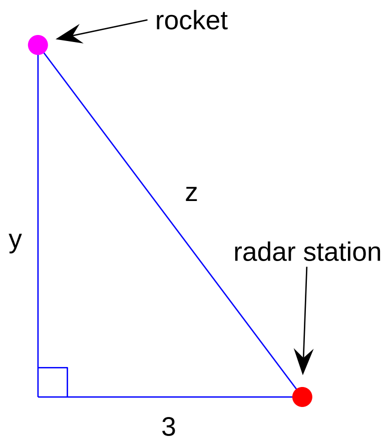
16. Let $f(x) = x^7 - x - 2$. Then $f'(x) = 7x^6 - 1$. Given $x_1 = 1$, Newton's method yields

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 1 - \frac{-2}{6} = 1 + \frac{1}{3} = 1\frac{1}{3} \text{ or } \frac{4}{3}. \end{aligned}$$

17. The quadratic approximation of $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$ near 27 is $Q(x) = \frac{f(27)}{0!} + \frac{f'(27)}{1!}(x-27)^1 + \frac{f''(27)}{2!}(x-27)^2$. Now $f'(x) = \frac{1}{3}x^{-2/3}$ and $f''(x) = -\frac{2}{9}x^{-5/3}$. Thus

$$\begin{aligned} Q(x) &= 3 + \frac{1}{27}(x-27) - \frac{2}{2 \cdot 3^2 \cdot 3^5}(x-27)^2 \\ &= 3 + \frac{1}{27}(x-27) - \frac{1}{2187}(x-27)^2. \end{aligned}$$

18. Here is a diagram depicting the situation.



- Place the origin directly beneath the rocket on the ground and the radar station 3 miles from the origin along the positive x-axis.
- Let z be the distance (through the air) between the rocket and the radar station. By the Pythagorean Theorem, $y^2 + 3^2 = z^2$. Differentiate with respect to t.

$$\begin{aligned} 2y\frac{dy}{dt} &= 2z\frac{dz}{dt} \\ \frac{dy}{dt} &= \frac{z\frac{dz}{dt}}{y} \end{aligned}$$

- When $t = 3$ seconds, we have $z = 5$, $\frac{dz}{dt} = 5000$, and

$$y = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4.$$

Therefore,

$$\begin{aligned} \frac{dy}{dt} &= \frac{(5)(5000)}{4} \\ &= 6250 \text{ mi/hr}. \end{aligned}$$