Fall 2007 Math 151 Common Exam 1A Thu, 27/Sep/2007

Name (print): _____

Signature: _____

Instructor: _____

Section # _____

Seat # _____

For official use only!

QN	PTS
1–12	
13	
14	
15	
16	
17	
Total	

Instructions

1. In **Part 1** (Problems 1–12), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.

2. Be sure to write your **name**, **section** number, and **version** of the exam (**1A** or **1B**) on your ScanTron.

3. In **Part 2** (Problems 13–17), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it.

4. Neither calculators nor computers are permitted on this exam.

5. Please turn off all cell phones so as not to interrupt other students.

Part 1: Multiple Choice (48 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 2t^2$. Find its average velocity (in m/s) over the time interval [1.0, 1.1].

   (a) 8.00
   (b) 5.80
   (c) 7.56
   (d) 6.00
   (e) -4.00

2. Find $\text{proj}_{\mathbf{a}}\mathbf{b}$, the vector projection of $\mathbf{b} = \langle -4, 1\rangle$ onto $\mathbf{a} = \langle 1, 2\rangle$.

   (a) $\langle -3, 3\rangle$
   (b) $\left\langle \frac{8}{17}, -\frac{2}{17} \right\rangle$
   (c) $\langle -5, -1\rangle$
   (d) $\left\langle -\frac{2}{5}, -\frac{4}{5} \right\rangle$
   (e) $\langle 5, 1\rangle$

3. Determine the limit $\lim\limits_{x\to-\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1}$.

   (a) 3
   (b) -9
   (c) -3
   (d) $-\infty$
   (e) 9

4. Find all vertical asymptotes to the graph of $y = \dfrac{x^2 - 2x}{x^2 - x - 2}$.

   (a) $x = 1$ and $x = -2$
   (b) $x = 1$ only
   (c) $x = -1$ only
   (d) $x = 2$ and $x = -1$
   (e) There are none.

5. Compute the limit $\lim\limits_{x \to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$.

   (a) $\dfrac{1}{4}$
   (b) $\dfrac{1}{128}$
   (c) $\dfrac{1}{32}$
   (d) $\dfrac{1}{8}$
   (e) $\dfrac{1}{16}$

6. A particle's motion in the xy-plane is given by $x = 2\sin t$, $y = 4 + \cos t$, $0 \leq t \leq 2\pi$. Describe its motion as t increases.

   (a) parabola traversed left-to-right
   (b) ellipse traversed clockwise
   (c) circle traversed counterclockwise
   (d) ellipse traversed counterclockwise
   (e) circle traversed clockwise

7. Let $\mathbf{a} = 5\mathbf{i} - 12\mathbf{j}$ and $\mathbf{b} = -3\mathbf{i} - 6\mathbf{j}$. Find the magnitude of the vector $\mathbf{a} - \mathbf{b}$.

   (a) $\sqrt{328}$
   (b) 2
   (c) $\sqrt{40}$
   (d) 10
   (e) 57

8. Find a unit vector that is parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

   (a) $\frac{1}{\sqrt{5}}\,\mathbf{i} + \frac{2}{\sqrt{5}}\,\mathbf{j}$
   (b) $\mathbf{j}$
   (c) $\frac{2}{5}\,\mathbf{i} + \frac{4}{5}\,\mathbf{j}$
   (d) $\mathbf{i}$
   (e) $\frac{1}{\sqrt{17}}\,\mathbf{i} + \frac{4}{\sqrt{17}}\,\mathbf{j}$

9. A tow truck drags a stalled car along a road. The chain makes an angle of 30° with the road and the tension in the chain is 1500 newtons. How much work (in joules) is done by the truck in pulling the car 1000 meters?

   (a) $750{,}000\sqrt{3}$
   (b) $1{,}500{,}000\sqrt{2}$
   (c) 750,000
   (d) $750{,}000\sqrt{2}$
   (e) 1,500,000

10. Find the limit $\lim_{x \to 0.5^-} \dfrac{2x - 1}{|2x^3 - x^2|}$.

(a) 0

(b) -1

(c) -4

(d) $-\infty$

(e) $+\infty$

11. In which interval does the equation $x + \cos x = 3$ have a solution?

(a) $(0, \pi)$

(b) $(-2\pi, -\pi)$

(c) $(2\pi, 3\pi)$

(d) $(-\pi, 0)$

(e) $(\pi, 2\pi)$

12. Find an equation of the tangent line to the curve $f(x) = 4x^2 - x^3$ at $x = 3$.
Write your answer in slope-intercept form.

(a) $y = -3x + 18$

(b) $y = 8x - 3x^2$

(c) $y = 3x + 6$

(d) $y = \left(4x^2 - x^3\right) x + 3$

(e) $y = 3x$

Part 2: Work-Out Problems (52 points)

Partial credit is possible. *SHOW ALL STEPS!*

13. Differentiate each of the following functions. You do NOT need to simplify.

    (a) [5 points] $g(t) = \left(2t^3 - 4t^{3/4} + 8t - 7\right)\left(t^4 + t^{1/3} + 45\right)$

    (b) [5 points] $q(x) = \dfrac{5x+1}{8x^3 + 4x - 2}$

14. The piecewise function f is defined as follows.

$$f(x) = \begin{cases} 2|x-2| & \text{if } x < 2 \\ (x-3)^2 - 1 & \text{if } 2 \le x \le 4 \\ 2x - 4 & \text{if } x > 4 \end{cases}$$

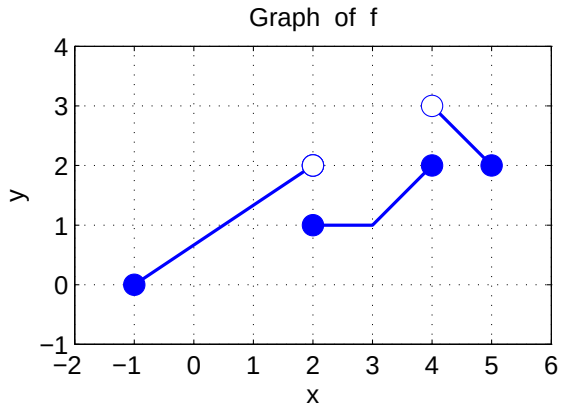
    (a) [5 points] Where is f discontinuous? Justify your answer.

    (b) [5 points] Is f differentiable at $x = 2$? If so, give the value of $f'(2)$ and write down the details of its computation. If not, explain why the derivative does not exist at $x = 2$. In either case, *justify your answer*.

15. [10 points] Let $f(x) = \sqrt{1+2x}$. Compute $f'(a)$ via the definition of derivative.

16. [10 points] Compute the derivative of $g(x) = \left|x^2 - 4\right|$ and state the domain of the derivative.

17. [12 points] Let $g(x) = x^2$. The graph of $f(x)$ appears below.



Give the values of the following limits (or explain why they do not exist).

(a) $\lim_{x \to 2^-} f(x) =$

(b) $\lim_{x \to 2^+} g(f(x)) =$

(c) $\lim_{x \to 2^+} f(g(x)) =$