Fall 2007 Math 151 Common Exam 2A Thu, 25/Oct/2007

Name (LAST, First): ______________________________

Signature: ______________________________

Instructor: ______________________________

Section # ______________________________

Seat # ______________________________

For official use only!

QN	PTS	MAX
1–14		56
15		8
16		8
17		8
18		8
19		8
20		8
Total		104

Instructions

1. In **Part 1** (Problems 1–14), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.
2. Be sure to write your **name**, **section** number, and **version** of the exam (**2A** or **2B**) on your ScanTron.
3. In **Part 2** (Problems 15–20), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it!
4. Neither calculators nor computers are permitted on this exam.
5. Please turn off all cell phones so as not to interrupt other students.

Part 1: Multiple Choice (56 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. Find the slope of the tangent line to $y = \sec x - 2\cos x$ at $x = \pi/3$.

   (a) $\sqrt{3}$
   (b) $2\sqrt{3}$
   (c) $3\sqrt{3}$
   (d) 1
   (e) 0

2. A table of values for f, g, f', and g' is given below. If $H(x) = g(f(x))$, find $H'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

   (a) 30
   (b) 65
   (c) 24
   (d) 63
   (e) 36

3. Determine the limit $\lim_{\theta \to 0} \dfrac{\sin(\cos\theta)}{\sec\theta}$.

   (a) 0
   (b) $-\cos 1$
   (c) $\sin 1$
   (d) $\cos 1$
   (e) $-\sin 1$

4. Let $f(x) = \sqrt{1 + xe^{-2x}}$. Compute $f'(0)$.

(a) 0
(b) 1/4
(c) 1/2
(d) 3/4
(e) 1

5. Find a Cartesian equation of the tangent line to the parametric curve $x = \cos t + \cos 2t$, $y = \sin t + \sin 2t$ for $t = \pi/2$.

(a) $y = -x$
(b) $y = 2x + 3$
(c) $y = 3x + 4$
(d) $y = x$
(e) $y = x + 2$

6. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 3 cm/s. When the length is 20 cm and the width is 10 cm, how fast (in cm^2/s) is the area of the rectangle increasing?

(a) 24
(b) 140
(c) 190
(d) 224
(e) 4800

7. A particle moves along the curve $y = \sqrt{1 + x^3}$. As it reaches the point (2, 3), the y-coordinate is increasing at a rate of 4 cm/s. How fast (in cm/s) is the x-coordinate of the point changing at that instant?

   (a) 1
   (b) 2
   (c) 3
   (d) 4
   (e) 5

8. Use the linear approximation with $a = 64$ to estimate the number $\sqrt[3]{70}$.

   (a) $4\frac{3}{4}$
   (b) $4\frac{1}{2}$
   (c) $4\frac{1}{4}$
   (d) $4\frac{1}{8}$
   (e) $4\frac{1}{16}$

9. Determine the quadratic approximation of $f(x) = \cos(x + \pi)$ at $a = 0$.

   (a) 1
   (b) $1 - \frac{1}{2}x^2$
   (c) $-1 + x^2$
   (d) $\sin\left(x - \frac{\pi}{2}\right)$
   (e) $-1 + \frac{1}{2}x^2$

10. Use Newton's method with initial approximation $x_1 = 0$ to find x_2, the second approximation to a root of the equation $e^x - 4x - \sin x = 0$.

(a) $1/e$
(b) $1/3$
(c) $1/2$
(d) $1/4$
(e) $2/e$

11. Let $p(x) = ax^2 + bx + c$, where a, b, c are constants. Given that $p(1) = 8,\ p'(1) = 4$, and $p''(1) = 6$, what is $p\left(\frac{1}{2}\right)$?

(a) $3\frac{1}{2}$
(b) $4\frac{1}{2}$
(c) $6\frac{3}{4}$
(d) 8
(e) $12\frac{1}{4}$

12. Let $g(x) = f^{-1}(x)$ be the inverse of $f(x) = 3 + x^2 + \tan(\pi x/2)$, $-1 < x < 1$. Find $g'(3)$.

(a) 0
(b) $4/\pi$
(c) 1
(d) $2/\pi$
(e) $\pi/4$

13. Evaluate $\lim_{x\to 3^+} \left(\frac{1}{2}\right)^{\frac{x}{x-3}}$.

(a) 0

(b) 1/2

(c) 1

(d) $e/2$

(e) ∞

14. Let $y = \frac{1-\sqrt{x}}{1+\sqrt{x}}$. Express x in terms of y.

(a) $x = \frac{\sqrt{y}-1}{\sqrt{y}+1}$

(b) $x = \frac{(1+y)^2}{(1-y)^2}$

(c) $x = \frac{1+\sqrt{y}}{1-\sqrt{y}}$

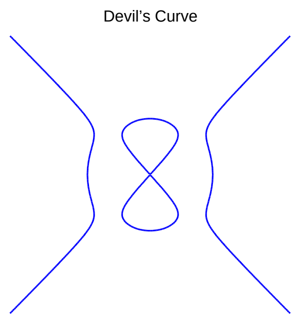
(d) $x = \frac{(1-y)^2}{(1+y)^2}$

(e) $x = \frac{1+\frac{1}{\sqrt{y}}}{1-\frac{1}{\sqrt{y}}}$

Part 2: Work-Out Problems (48 points)

Partial credit is possible. Present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it!

15. [8 points] Find an equation of the tangent line to the "devil's curve" $y^4 - 4y^2 = x^4 - 5x^2$ at $\left(\sqrt{5}, 2\right)$.



16. A particle's position in the plane at time t is given by the vector function $\mathbf{r}(t) = \langle t\cos t, t\sin t\rangle$. For time $t = \pi$, compute the following items regarding the particle. Show all work!

(a) [2 points] position:

(b) [2 points] velocity:

(c) [2 points] speed:

(d) [2 points] acceleration:

17. [8 points] Find all values of t for which the tangent line to the curve $x = 2t^3 + 3t^2 - 12t$, $y = 2t^3 + 3t^2 + 1$ is horizontal or vertical. When you have finished, fill in the following table. For "type," write H for horizontal and V for vertical. (Add additional rows if necessary.)

t-value	type

18. [8 points] The circumference of a sphere (the length of its "equatorial circle") was measured to be 20 cm with a possible error of 1 cm. Estimate the maximum error in the calculated volume of the sphere using differentials. Recall that the volume of a sphere is $V = \frac{4}{3}\pi r^3$ and the circumference of a circle is $C = 2\pi r$. Here r is the radius.

19. [8 points] Solve the following two equations.

(a) $10\left(1+e^{-x}\right)^{-1}=3$

(b) $\log_2(2x+1)=2-\log_2(4x)$

20. [8 points] Two French seahorses frolic in the sea at Mediterranean Downs, practicing for the big race on Saturday! Phillipe travels due north of the starting post, while Gigi heads due east. At time t, Phillipe's distance (in cm) from the post is $y=7+2t+\frac{1}{2}t^2$, whereas that of Gigi is $x=6+4t$. Here time is measured in seconds. At what rate is the distance z between Phillipe and Gigi changing after $t=2$ seconds?