Fall 2007 Math 151 Common Exam 3A Tue, 27/Nov/2007

Name (LAST, First): ______________________________

Signature: ______________________________

Instructor: ______________________________

Section # ______________________________

Seat # ______________________________

For official use only!

QN	PTS	MAX
1–14		56
15		8
16		8
17		8
18		8
19		8
20		8
Total		104

Instructions

1. In **Part 1** (Problems 1–14), mark the correct choice on your ScanTron form using a No. 2 pencil. *For your own record, also mark your choices on your exam!* ScanTrons will be collected from all examinees **after 90 minutes** and will *not* be returned.
2. Be sure to write your **name**, **section** number, and **version** of the exam (**3A** or **3B**) on your ScanTron.
3. In **Part 2** (Problems 15–20), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it!
4. Neither calculators nor computers are permitted on this exam.
5. Please turn off all cell phones so as not to interrupt other students.

Part 1: Multiple Choice (56 points)

Read each question carefully. Each problem in Part 1 is worth **4 points**.

1. Find an equation of the tangent line to $f(x) = \ln\left(x^2 e^{x^3}\right)$ at $(1, 1)$. [NOTE: You may first simplify $f(x)$ by using properties of logarithms.]

   (a) $y = x$
   (b) $y = 3x - 2$
   (c) $y = 1$
   (d) $y = 5x - 4$
   (e) $y = 2x - 1$

2. Let $y = \left(\tan^{-1} x\right)^{\ln x}$. Compute $y'(1)$.

   (a) 1
   (b) $\ln(\pi/4)$
   (c) 0
   (d) $\pi/4$
   (e) e

3. The population of the world was 5 billion in 1990 and 6 billion in 2000. Assuming exponential growth, what will be the population in 2010?

   (a) 6.8 billion
   (b) 7.0 billion
   (c) 7.2 billion
   (d) 7.5 billion
   (e) 8.0 billion

4. Given $f(x) = \ln\left(\sec^2 x\right)$, compute $f'(x)$.

(a) $\ln(\tan x)$

(b) $2\sec x$

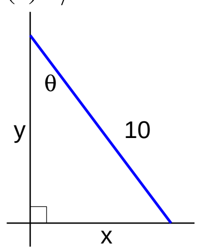
(c) $\ln(2\sec x)$

(d) 2

(e) $2\tan x$

5. A ladder 10 ft long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of 2 ft/s, how fast (in rad/s) is the angle θ between the ladder and the wall changing when the bottom of the ladder is 6 ft from the base of the wall?

(a) $1/4$ (b) $\pi/4$ (c) $1/8$ (d) $\pi/2$ (e) 1



6. Evaluate the limit $\lim_{x\to 1^+}\left(\dfrac{x}{x-1} - \dfrac{1}{\ln x}\right)$.

(a) ∞

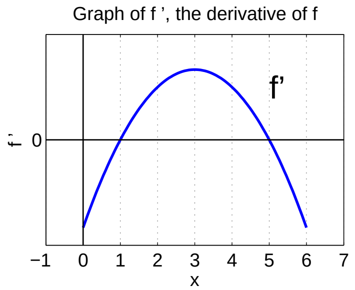
(b) 1

(c) $1/2$

(d) $1/3$

(e) $-\infty$

7. The graph of the *derivative* f' of a function f is shown. At what value of x (if any) does f have a local maximum?
(a) 0 (b) 1 (c) 3 (d) 5 (e) none



8. Find the value of the absolute minimum of $f(x) = x^3 - 6x^2 + 9x + 2$ on $[-1, 2]$.

(a) -1
(b) 2
(c) -14
(d) 6
(e) 1

9. Determine the value of x for which $f(x) = x + \cos 2x$ increases most rapidly on $[0, \pi]$.

(a) $3\pi/4$
(b) $\pi/12$
(c) $\pi/6$
(d) $\pi/4$
(e) $2\pi/3$

10. Find a number c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 + x - 1$ on the interval $[0, 2]$.

(a) 1
(b) $2/\sqrt{3}$
(c) $1/3$
(d) $\sqrt{3}$
(e) $3/2$

11. Which of the following is an antiderivative of $f(x) = \left(\frac{1}{x} + 2\ln x\right) e^{2x}$?

(a) $\left(\frac{2}{x} + \ln x\right) e^{2x}$
(b) $\frac{1}{2}\left(\frac{2}{x} - \frac{1}{x^2}\right) e^{2x}$
(c) $e^{2x} \ln x$
(d) $(\ln x + 2x \ln x - 2x) \cdot \frac{1}{2} e^{2x}$
(e) $\left(\frac{2}{x} - \frac{1}{x^2}\right) e^{2x} + 2\left(\frac{1}{x} + 2\ln x\right) e^{2x}$

12. A moving particle starts at an initial position $\mathbf{r}(0) = \langle 1, 0 \rangle$ with initial velocity $\mathbf{v}(0) = \langle 1, -1 \rangle$. Its acceleration at time t is $\mathbf{a}(t) = \langle 4t, 6t \rangle$. What is the particle's position at time $t = 3$?

(a) $\langle 10, 20 \rangle$
(b) $\langle 12, 18 \rangle$
(c) $\langle 18, 27 \rangle$
(d) $\langle 22, 24 \rangle$
(e) $\langle 21, 30 \rangle$

13. Estimate the value of the definite integral $\int_0^6 \frac{x-1}{x+1}\,dx$ using the Midpoint Rule with $n = 3$.

(a) 2
(b) $6 - 2\ln 7$
(c) $7/3$
(d) $15/6$
(e) $49/20$

14. Find the critical numbers of $f(x) = x^{2/3}(x-1)^2$.

(a) 1
(b) $0, \frac{1}{4}, 1$
(c) $0, 1$
(d) $0, \frac{2}{3}, 1$
(e) $0, \frac{3}{4}, 1$

Part 2: Work-Out Problems (48 points)

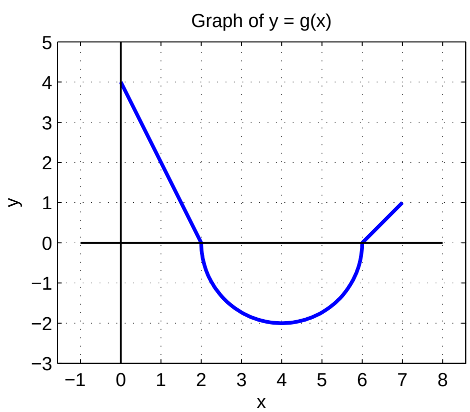
Partial credit is possible. Present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded not merely on the final answer, but also on the quality and correctness of the work leading up to it!

15. [8 points] Compute the limit $\lim_{x\to 0}\left(\cos\left(\frac{x}{3}\right)\right)^{-9/x^2}$.

16. [8 points] If 1200 cm^2 of material is available to make a box with a square base and no top, find the largest possible volume of the box.

17. [8 points] The acceleration of a particle that moves along the x-axis is given by $a(t) = 10\sin t + 3\cos t$. The positions of the particle at times $t = 0$ and $t = 2\pi$ are given by $x(0) = 0$ and $x(2\pi) = 12$, respectively. Find the position $x(t)$ of the particle at a general time t.

18. [8 points] The graph of g consists of two straight lines and a semicircle as illustrated below.



Use it to evaluate each of these three integrals.

(a) $\int_0^2 g(x)\,dx =$

(b) $\int_2^6 g(x)\,dx =$

(c) $\int_0^7 g(x)\,dx =$

19. [8 points] Let $f(x) = \sqrt{\tan x}$. Note that f is differentiable (and thus continuous) on $(0, \pi/2)$.

(a) Show that f is increasing on $(0, \pi/2)$.

(b) Determine m and M—the minimum and maximum values of f, respectively, on $[\pi/4, \pi/3]$.

(c) Now $\int_{\pi/4}^{\pi/3} m\,dx \le \int_{\pi/4}^{\pi/3} \sqrt{\tan x}\,dx \le \int_{\pi/4}^{\pi/3} M\,dx$. Accordingly, estimate $\int_{\pi/4}^{\pi/3} \sqrt{\tan x}\,dx$ by averaging $\int_{\pi/4}^{\pi/3} m\,dx$ and $\int_{\pi/4}^{\pi/3} M\,dx$.

20. [8 points] Find area of the largest rectangle that can be inscribed inside a circle of radius 1 cm.
Show all steps and justify all assertions!