# Math 151 Fall 2009 Exam I
# Solutions-Form A

1. A: The vector $\langle 2, -3 \rangle$ is parallel to the line $x = 2t+1$, $y = -3t+5$. Hence $\langle 3, 2 \rangle$ is perpendicular to the line. Divide by the magnitude to make the vector a unit vector. $\dfrac{\langle 3,2 \rangle}{\sqrt{13}} = \left\langle \dfrac{3}{\sqrt{13}}, \dfrac{2}{\sqrt{13}} \right\rangle$.

2. D: $\displaystyle \lim_{x \to 0} \frac{\sqrt{9-x}-3}{x} = \lim_{x \to 0} \frac{\sqrt{9-x}-3}{x} \frac{\sqrt{9-x}+3}{\sqrt{9-x}+3}$
$\displaystyle = \lim_{x \to 0} \frac{9-x-9}{x(\sqrt{9-x}+3)} = -\frac{1}{6}$.

3. E: Use the product rule to differentiate $H(x) = x^2 f(x)$: $H'(x) = x^2 f'(x) + 2x f(x)$. Since $f(-2) = 3$ and $f'(-2) = -4$,
$H'(-2) = (-2)^2 f'(-2) + 2(-2) f(-2)$
$= 4(-4) + (-4)(3) = -28$

4. D: $W = |\mathbf{F}||\mathbf{D}|\cos(\theta)$. Here, $|\mathbf{F}| = 8$ pounds, $|\mathbf{D}| = 25$ feet and $\theta = 60°$. Hence
$W =$ (8 pounds) (25 feet) $(\cos 60°)$
$= 100$ foot pounds.

5. B: Use the quotient rule to differentatate $f(x) = \dfrac{x}{1+x}$.
$f'(x) = \dfrac{(1)(1+x)-(x)(1)}{(1+x)^2} = \dfrac{1}{(1+x)^2}$. Hence the slope of the tangent line is $m = f'(2) = \dfrac{1}{9}$. Also, when $x = 2$, $y = \dfrac{2}{3}$. Thus the equation of the tangent line is $y - \dfrac{2}{3} = \dfrac{1}{9}(x-2)$.

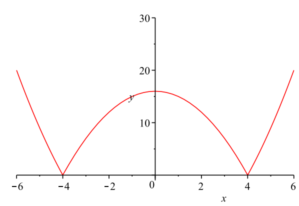
6. C: $x = \sin t$ and $y = 4 + \cos t$ is equivalent to $x = \sin t$, $y - 4 = \cos t$. Since $\sin^2 t + \cos^2 t = 1$, $x^2 + (y-4)^2 = 1$.

7. B: To test $f(x) = \begin{cases} 2x+1 & \text{if } x \le -1 \\ x^2 - 3 & \text{if } -1 < x \le 2 \\ \dfrac{1}{x} + \dfrac{1}{2} & \text{if } x > 2 \end{cases}$ for continuity, we first note that $2x+1$ is continuous for $x < -1$, $x^2 - 3$ is continuous for $-1 < x < 2$ and $\dfrac{1}{x} + \dfrac{1}{2}$ is continuous for $x > 2$. Thus we need only to check continuity for $x = -1$ and $x = 2$.
$\displaystyle \lim_{x \to -1^-} f(x) = -1$, $\displaystyle \lim_{x \to -1^+} f(x) = -2$.
Thus $\displaystyle \lim_{x \to -1} f(x)$ does not exist, hence $f(x)$ is not continuous at $x = -1$.
$\displaystyle \lim_{x \to 2^-} f(x) = 1$, $\displaystyle \lim_{x \to 2^+} f(x) = 1$, $f(2) = 1$, thus $f(x)$ is continuous at $x = 2$. The only place of discontunity is $x = -1$.

8. B: $\displaystyle \lim_{x \to -3^+} \frac{x-1}{x^2(x+3)} = -\infty$ since $x = -3$ is a vertical asymptote and $\dfrac{x-1}{x^2(x+3)} < 0$ for $x > -3$.

9. D: First, note $f(x) = x^3 - x^2 + 3x - 1$ is continuous, so we will apply the Intermediate Value Theorem. Since $f(0) = -1 < 0$ and $f(1) = 2 > 0$, there is a solution to $f(c) = 0$ on the interval $(0, 1)$.

10. C: $f(x) = |x^2 - 16|$ is not differentiable at $x = \pm 4$ since $f'(4)$ and $f'(-4)$ does not exist. (See figure below)




11. Multiply by the conjugate:
$\displaystyle \lim_{x \to -\infty} \left(x + \sqrt{x^2+3x}\right) \frac{x - \sqrt{x^2+3x}}{x - \sqrt{x^2+3x}}$
$\displaystyle = \lim_{x \to -\infty} \frac{-3x}{x - \sqrt{x^2+3x}}$. Divide the numerator and denominstor by $x$:
$\displaystyle = \lim_{x \to -\infty} \frac{-3x/x}{x/x - (\sqrt{x^2+3x})/x}$
$\displaystyle = \lim_{x \to -\infty} \frac{-3}{1 + \sqrt{1+3/x}} = -\frac{3}{2}$

12. (i) To find the cosine of the angle between the vectors $\langle 1,4\rangle$ and $\langle 2,3\rangle$, we will use the formula $\cos\theta = \dfrac{\langle 1,4\rangle \cdot \langle 2,3\rangle}{(|\langle 1,4\rangle|)(|\langle 2,3\rangle|)}$

$\cos\theta = \dfrac{2+12}{\sqrt{17}\sqrt{13}} = \dfrac{14}{\sqrt{221}}$.

(ii) Let $\mathbf{a} = \langle 2,3\rangle$ and $\mathbf{b} = \langle 1,4\rangle$ Then

$\text{comp}_{\mathbf{a}}\mathbf{b} = \dfrac{\langle 2,3\rangle \cdot \langle 1,4\rangle}{|\langle 2,3\rangle|} = \dfrac{14}{\sqrt{13}}$

(iii) $\text{proj}_{\mathbf{a}}\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$

$= \dfrac{\langle 2,3\rangle \cdot \langle 1,4\rangle}{|\langle 2,3\rangle|^2}\langle 2,3\rangle$

$= \dfrac{14}{13}\langle 2,3\rangle$

$= \left\langle \dfrac{28}{13}, \dfrac{42}{13}\right\rangle$

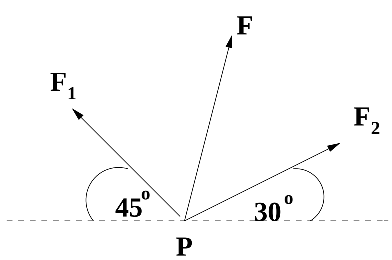
13. $f(x) = \dfrac{2}{x-3}$. $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$= \lim\limits_{h\to 0}\dfrac{\dfrac{2}{x+h-3} - \dfrac{2}{x-3}}{h}$

$= \lim\limits_{h\to 0}\dfrac{2(x-3)-2(x+h-3)}{h(x+h-3)(x-3)}$

$= \lim\limits_{h\to 0}\dfrac{-2h}{h(x+h-3)(x-3)}$

$= \dfrac{-2}{(x-3)^2}$

14. 


Since $|\mathbf{F_1}| = 8$ pounds and $|\mathbf{F_2}| = 10$ pounds,

$\mathbf{F_1} = \langle -8\cos 45°, 8\sin 45°\rangle = \left\langle -4\sqrt{2}, 4\sqrt{2}\right\rangle$.

$\mathbf{F_2} = \langle 10\cos 30°, 10\sin 30°\rangle = \left\langle 5\sqrt{3}, 5\right\rangle$. Thus the resultant force is

$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2} = \left\langle -4\sqrt{2}+5\sqrt{3}, 4\sqrt{2}+5\right\rangle$. Thus the magnitude of the resultant force is

$|\mathbf{F}| = \sqrt{\left(-4\sqrt{2}+5\sqrt{3}\right)^2 + \left(4\sqrt{2}+5\right)^2}$ pounds

15. a.) Since $x < 2$, $|x-2| = -(x-2)$. Thus $\lim\limits_{x\to 2^-}\dfrac{x-2}{|x-2|} = \lim\limits_{x\to 2^-}\dfrac{x-2}{-(x-2)}$
$= -1$

b.) To find the value of $a$ for which $\lim\limits_{x\to 1}\dfrac{3x^2+ax+a+3}{x^2+x-2}$ exists, we use the fact that, in order for the limit to exist at $x=1$, $3x^2+ax+a+3$ must have a factor of $x-1$ in order to eliminate the division by zero. Hence if $x-1$ is a factor of $3x^2+ax+a+3$, it follows that

$3(1)^2 + a(1) + a + 3 = 0$. Thus $3+a+a+3 = 0$ yielding $a = -3$.