

Math 151 Fall 2009 Exam I Solutions-Form B

1. D: The vector $\langle 3, 2 \rangle$ is parallel to the line $x = 3t+1, y = 2t+5$. Hence $\langle 2, -3 \rangle$ is perpendicular to the line. Divide by the magnitude to make the vector a unit vector. $\frac{\langle 2, -3 \rangle}{\sqrt{13}} = \left\langle \frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}} \right\rangle$.

$$2. \text{ A: } \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x} = \lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x} \frac{\sqrt{9+x} + 3}{\sqrt{9+x} + 3} \\ = \lim_{x \rightarrow 0} \frac{9+x-9}{x(\sqrt{9+x}+3)} = \frac{1}{6}.$$

3. C: Use the product rule to differentiate $H(x) = x^2 f(x)$: $H'(x) = x^2 f'(x) + 2x f(x)$. Since $f(2) = 4$ and $f'(2) = -3$, $H'(2) = (2)^2 f'(2) + 2(2)f(2) = 4(-3) + (4)(4) = 4$

4. A: $W = |\mathbf{F}||\mathbf{D}| \cos(\theta)$. Here, $|\mathbf{F}| = 8$ pounds, $|\mathbf{D}| = 25$ feet and $\theta = 30^\circ$. Hence $W = (8 \text{ pounds})(25 \text{ feet})(\cos 30^\circ) = 100\sqrt{3}$ foot pounds.

5. B: Use the quotient rule to differentiate $f(x) = \frac{x}{1+x}$. $f'(x) = \frac{(1)(1+x) - (x)(1)}{(1+x)^2} = \frac{1}{(1+x)^2}$. Hence the slope of the tangent line is $m = f'(2) = \frac{1}{9}$. Also, when $x = 2, y = \frac{2}{3}$. Thus the equation of the tangent line is $y - \frac{2}{3} = \frac{1}{9}(x - 2)$.

6. B: $x = 4 + \sin t$ and $y = \cos t$ is equivalent to $x - 4 = \sin t, y = \cos t$. Since $\sin^2 t + \cos^2 t = 1$, $(x - 4)^2 + y^2 = 1$.

7. A: To test $f(x) = \begin{cases} 2x+1 & \text{if } x \leq -1 \\ x^2-2 & \text{if } -1 < x \leq 2 \\ \frac{1}{x} + \frac{1}{2} & \text{if } x > 2 \end{cases}$ for continuity, we first note that $2x+1$ is continuous for $x < -1$, x^2-2 is continuous for $-1 < x < 2$ and $\frac{1}{x} + \frac{1}{2}$ is continuous for $x > 2$. Thus we need

only to check continuity for $x = -1$ and $x = 2$. $\lim_{x \rightarrow -2^-} f(x) = 2, \lim_{x \rightarrow -2^+} f(x) = 1$.

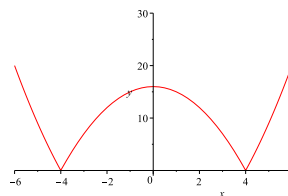
Thus $\lim_{x \rightarrow 2} f(x)$ does not exist, hence $f(x)$ is not continuous at $x = 2$.

$\lim_{x \rightarrow -1^-} f(x) = -1, \lim_{x \rightarrow -1^+} f(x) = -1, f(-1) = -1$, thus $f(x)$ is continuous at $x = -1$. The only place of discontinuity is $x = 2$.

8. C: $\lim_{x \rightarrow -3^-} \frac{x-1}{x^2(x+3)} = \infty$ since $x = -3$ is a vertical asymptote and $\frac{x-1}{x^2(x+3)} > 0$ for $x < -3$.

9. E: First, note $f(x) = x^3 + x^2 - 3x - 10$ is continuous, so we will apply the Intermediate Value Theorem. Since $f(2) = -4 < 0$ and $f(3) = 17 > 0$, there is a solution to $f(c) = 0$ on the interval $(2, 3)$.

10. C: $f(x) = |x^2 - 16|$ is not differentiable at $x = \pm 4$ since $f'(4)$ and $f'(-4)$ does not exist. (See figure below)



11. Multiply by the conjugate:

$$\lim_{x \rightarrow -\infty} \left(x + \sqrt{x^2 + 5x} \right) \frac{x - \sqrt{x^2 + 5x}}{x - \sqrt{x^2 + 5x}} \\ = \lim_{x \rightarrow -\infty} \frac{-5x}{x - \sqrt{x^2 + 5x}}. \text{ Divide the numerator and denominator by } x: \\ = \lim_{x \rightarrow -\infty} \frac{-5x/x}{x/x - (\sqrt{x^2 + 5x})/x} \\ = \lim_{x \rightarrow -\infty} \frac{-5}{1 + \sqrt{1 + 5/x}} = -\frac{5}{2}$$

12. (i) To find the cosine of the angle between the vectors $\langle 1, 2 \rangle$ and $\langle 3, 4 \rangle$, we will use the formula

$$\cos \theta = \frac{\langle 1, 2 \rangle \cdot \langle 3, 4 \rangle}{(|\langle 1, 2 \rangle|)(|\langle 3, 4 \rangle|)}$$

$$\cos \theta = \frac{3 + 8}{\sqrt{5}\sqrt{25}} = \frac{11}{5\sqrt{5}}.$$

- (ii) Let $\mathbf{a} = \langle 3, 4 \rangle$ and $\mathbf{b} = \langle 1, 2 \rangle$. Then

$$\text{comp}_{\mathbf{a}} \mathbf{b} = \frac{\langle 3, 4 \rangle \cdot \langle 1, 2 \rangle}{|\langle 3, 4 \rangle|} = \frac{11}{5}$$

$$\text{(iii) } \text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} \mathbf{a}$$

$$= \frac{\langle 3, 4 \rangle \cdot \langle 1, 2 \rangle}{|\langle 3, 4 \rangle|^2} \langle 3, 4 \rangle$$

$$= \left\langle \frac{33}{25}, \frac{44}{25} \right\rangle$$

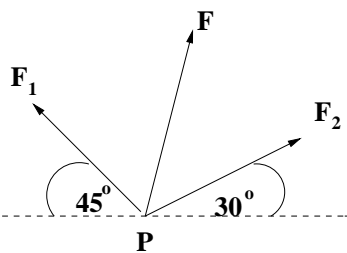
13. $f(x) = \frac{3}{x-2}$. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\frac{3}{x+h-2} - \frac{3}{x-2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x-2) - 3(x+h-2)}{h(x+h-2)(x-2)}$$

$$= \lim_{h \rightarrow 0} \frac{-3h}{h(x+h-2)(x-2)}$$

$$= \frac{-3}{(x-2)^2}$$



14.

Since $|\mathbf{F}_1| = 10$ pounds and $|\mathbf{F}_2| = 8$ pounds,

$$\mathbf{F}_1 = \langle -10 \cos 45^\circ, 10 \sin 45^\circ \rangle = \langle -5\sqrt{2}, 5\sqrt{2} \rangle.$$

$\mathbf{F}_2 = \langle 8 \cos 30^\circ, 8 \sin 30^\circ \rangle = \langle 4\sqrt{3}, 4 \rangle$. Thus the resultant force is

$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \langle -5\sqrt{2} + 4\sqrt{3}, 5\sqrt{2} + 4 \rangle$. Thus the magnitude of the resultant force is

$$|\mathbf{F}| = \sqrt{(-5\sqrt{2} + 4\sqrt{3})^2 + (5\sqrt{2} + 4)^2} \text{ pounds}$$

15. a.) Since $x < 2$, $|x - 2| = -(x - 2)$. Thus

$$\lim_{x \rightarrow 2^-} \frac{x-2}{|x-2|} = \lim_{x \rightarrow 2^-} \frac{x-2}{-(x-2)} = -1$$

- b.) To find the value of a for which $\lim_{x \rightarrow 1} \frac{3x^2 + ax + a + 5}{x^2 + x - 2}$ exists, we use the fact that, in

order for the limit to exist at $x = 1$, $3x^2 + ax + a + 5$ must have a factor of $x - 1$ in order to eliminate the division by zero. Hence if $x - 1$ is a factor of $3x^2 + ax + a + 5$, it follows that

$3(1)^2 + a(1) + a + 5 = 0$. Thus $3 + a + a + 5 = 0$ yielding $a = -4$.