Math 151 Fall 2009 Exam II Solutions-Form A

1. D: $\lim_{x\to 2} \frac{\sin(x-2)}{x^2+x-6} = \lim_{x\to 2} \frac{\sin(x-2)}{(x+3)(x-2)}$ $= \lim_{x\to 2} \frac{\sin(x-2)}{(x-2)} \frac{1}{x+3} = \frac{1}{5}$

2. B: $f(x) = e^{x^2}$. By the chain rule, $f'(x) = 2xe^{x^2}$. By the product and chain rule, $f''(x) = 4x^2e^{x^2} + 2e^{x^2}$. Hence $f''(1) = 4e + 2e = 6e$.

3. D: First we will find the tangent vector at $t = \frac{\pi}{6}$ and then make it a unit vector by dividing by the magnitude:

   $\mathbf{r}(t) = \langle 4\cos t, 2\sin t\rangle$ thus

   $\mathbf{r}'(t) = \langle -4\sin t, 2\cos t\rangle$. Therefore

   $\mathbf{r}'\left(\frac{\pi}{6}\right) = \left\langle -2, \sqrt{3}\right\rangle$. The unit vector is $\frac{\left\langle -2, \sqrt{3}\right\rangle}{|\left\langle -2, \sqrt{3}\right\rangle|} = \left\langle \frac{-2}{\sqrt{7}}, \frac{\sqrt{3}}{\sqrt{7}}\right\rangle = \left\langle -\frac{2}{\sqrt{7}}, \sqrt{\frac{3}{7}}\right\rangle$.

4. A: $h(x) = xf(x^3)$, thus by the product and chain rule, $h'(x) = f(x^3) + 3x^3f'(x^3)$. Thus

   $h'(2) = f(8) + (24)f'(8) = 3 - 24 = -21$.

5. A: $f(x) = \cos(2x)$. The quadratic approximation for $f(x)$ at $x = 0$ is

   $Q(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2$. Now $f'(0) = \cos(0) = 1$. $f'(0) = -2\sin(0) = 0$,

   $f''(0) = -4\cos(0) = -4$. Hence $Q(x) = 1 - 2x^2$.

6. B: $\lim_{x\to -3^+} e^{1/(x+3)} = e^{\lim_{x\to -3^+} 1/(x+3)} = e^{\infty} = \infty$

7. E: $x = \sqrt{t}$, $y = t^2 + 5$. The parameter $t = 4$ corresponds to the point $(2, 21)$. Thus $m = \frac{dy/dt}{dx/dt}$ evaluated at $t = 4$.

   $m = \frac{2t}{\frac{1}{2\sqrt{t}}} = 4t\sqrt{t}$. Thus when $t = 4$, $m = 32$.

8. C: $s(t) = \sin t + \frac{1}{4}t^2$. We want to solve $a(t) = 0$, where $a(t)$ is the acceleration. Now,

   $s(t) = \sin t + \frac{1}{4}t^2$, thus $v(t) = \cos t + \frac{1}{2}t$, so

   $a(t) = -\sin t + \frac{1}{2}$. Now, $a(t) = 0$ if $\sin t = \frac{1}{2}$, and this is true if $t = \frac{\pi}{6}$ or $t = \frac{5\pi}{6}$.

9. B: $x^3 + y^3 = 6xy$. Differentiating implicitly,

   $3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$. Substitute $x = 3$, $y = 3$ and solve for $\frac{dy}{dx}$. $3(9) + 3(9)\frac{dy}{dx} = 6(3) + 6(3)\frac{dy}{dx}$. $27 + 27\frac{dy}{dx} = 18 + 18\frac{dy}{dx}$, thus $9 = -9\frac{dy}{dx}$, hence $\frac{dy}{dx} = -1$

10. C: $s(t) = t^2 - 2t + 3$. To find the total distance traveled in the first 3 seconds, we must first find where the velocity is positive and negative. $v(t) = 2t - 2$. Thus the particle changes direction at $t = 1$, and more specifically, the particle is moving in the negative direction between $t = 0$ and $t = 1$ and the particle is moving in the positive direction between $t = 1$ and $t = 3$. Hence the total distance traveled in the first 3 seconds is $|s(1) - s(0)| + s(3) - s(1)$

    $= |-1| + 4 = 5$ feet.

11. A: The tangent line is horizontal when $\frac{dy}{dt} = 0$ and $\frac{dx}{dt} \neq 0$. Now $x = t^2 - 2t + 4$ and $y = t^3 - 3t^2$. $\frac{dy}{dt} = 3t^2 - 6t = 3t(t-2)$. $\frac{dy}{dt} = 0$ if $t = 0$ or $t = 2$. Note that $\frac{dx}{dt} = 2t - 2 \neq 0$ at these two values of t. Now if $t = 0$, $x = 4$ and $y = 0$. If $t = 2$, $x = 4$ and $y = -4$. Hence the points are $(4, 0)$ and $(4, -4)$. .

12. (i) $f(x) = x\cos^3(x^2)$. By the product and chain rule,

$f'(x) = (x)\left(3\cos^2(x^2)(-\sin(x^2))(2x)\right) + \cos^3(x^2)$

$= -6x^2\cos^2(x^2)\sin(x^2) + \cos^3(x^2)$.

(ii) $g(t) = \sqrt[3]{4t - t^2} = (4t - t^2)^{1/3}$. By the chain rule,

$g'(t) = \frac{1}{3}(4t - t^2)^{-2/3}(4 - 2t)$

$= \frac{4 - 2t}{3(4t - t^2)^{2/3}}$

(iii) $h(x) = e^{\tan\sqrt{x}}$, thus by the chain rule,

$h'(x) = e^{\tan\sqrt{x}}\sec^2\sqrt{x}(\frac{1}{2\sqrt{x}})$

13. Let A be the area, h be the height, and b be the base of the triangle at time t.

We are given $\frac{dh}{dt} = -2$ cm/min and $\frac{dA}{dt} = \frac{1}{2}$ cm^2/min. We want to find $\frac{db}{dt}$ when $h = 6$ cm

and $A = 60$ cm^2. The Area of a triangle is $A = \frac{1}{2}bh$. Differentiate implicitly with respect to time using the product rule:

$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt}h + \frac{1}{2}b\frac{dh}{dt}$. Now, when $h = 6$ cm and $A = 60$ cm^2, $b = 20$ cm. Hence if we substitute $\frac{dh}{dt} = -2$cm/min, $\frac{dA}{dt} = \frac{1}{2}$ cm^2/min, $h = 6$ cm, $A = 60$ cm^2 and $b = 20$ cm, we obtain

$\frac{1}{2} = \frac{1}{2}\frac{db}{dt}(6) + \frac{1}{2}(20)(-2)$. Thus $\frac{db}{dt} = \frac{41}{6}$ cm/min.

14. To find $\frac{dy}{dx}$, or equivalently, y', differentiate

$\sin(7y + 5x) = 3x^2 + y^3$ implicitly with respect to x using the product rule and chain rule:

$\cos(7y + 5x)(7y' + 5) = 6x + 3y^2y'$

$7\cos(7y + 5x)y' + 5\cos(7y + 5x) = 6x + 3y^2y'$

$y'(7\cos(7y + 5x) - 3y^2) = 6x - 5\cos(7y + 5x)$

$y' = \frac{6x - 5\cos(7y + 5x)}{7\cos(7y + 5x) - 3y^2}$.

15. Recall if g is the inverse of f, then

$g'(a) = \frac{1}{f'(g(a))}$. Thus $g'(2) = \frac{1}{f'(g(2))}$.

Since $f(0) = 2$, $g(2) = 0$. $g'(2) = \frac{1}{f'(0)}$. Now $f'(x) = 2e^{2x} + 4$, hence $f'(0) = 6$.

Therefore $g'(2) = \frac{1}{f'(0)} = \frac{1}{6}$

16. $y = \frac{2x + 1}{4 - x}$. To find $f^{-1}(x)$, first interchange x and y:

$x = \frac{2y + 1}{4 - y}$

$x(4 - y) = 2y + 1$

$4x - xy = 2y + 1$

$4x - 1 = y(2 + x)$, thus $f^{-1}(x) = \frac{4x - 1}{2 + x}$

17. (a) The linear approximation for $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$. Here, $a = 9$. hence $L(x) = f(9) + f'(9)(x - 9)$. Now $f(x) = \sqrt{x}$, thus $f(9) = 3$. $f'(x) = \frac{1}{2\sqrt{x}}$, thus $f'(9) = \frac{1}{6}$.

$L(x) = 3 + \frac{1}{6}(x - 9)$, or $L(x) = \frac{1}{6}x + \frac{3}{2}$.

(b) Now, $f(x) \approx L(x)$ for x near a. Thus

$\sqrt{x} \approx \frac{1}{6}x + \frac{3}{2}$ for x near 9.

Thus $\sqrt{9.1} \approx \frac{1}{6}(9.1) + \frac{3}{2} = \frac{181}{60}$