Math 151 Fall 2009 Exam II Solutions-Form B

1. B: $\lim_{x\to 2} \frac{\sin(x-2)}{x^2+2x-8} = \lim_{x\to 2} \frac{\sin(x-2)}{(x+4)(x-2)}$
$= \lim_{x\to 2} \frac{\sin(x-2)}{(x-2)} \frac{1}{x+4} = \frac{1}{6}$

2. D: $f(x) = e^{-x^2}$.
By the chain rule, $f'(x) = -2xe^{-x^2}$. By the product and chain rule,
$f''(x) = 4x^2e^{-x^2} - 2e^{-x^2}$. Hence
$f''(1) = 4e^{-1} - 2e^{-1} = 2e^{-1} = \frac{2}{e}$.

3. C: First we will find the tangent vector at $t = \frac{\pi}{3}$ and then make it a unit vector by dividing by the magnitude:
$\mathbf{r}(t) = \langle 4\cos t, 2\sin t\rangle$ thus
$\mathbf{r}'(t) = \langle -4\sin t, 2\cos t\rangle$. Therefore
$\mathbf{r}'\left(\frac{\pi}{3}\right) = \left\langle -2\sqrt{3}, 1\right\rangle$.
The unit vector is $\frac{\left\langle -2\sqrt{3}, 1\right\rangle}{|\left\langle -2\sqrt{3}, 1\right\rangle|}$
$= \left\langle -\frac{2\sqrt{3}}{\sqrt{13}}, \frac{1}{\sqrt{13}}\right\rangle$.

4. B: $h(x) = xf(x^3)$, thus by the product and chain rule, $h'(x) = f(x^3) + 3x^3f'(x^3)$. Thus
$h'(2) = f(8) + (24)f'(8) = 3 - 24 = -21$.

5. D: $f(x) = \cos(2x)$. The quadratic approximation for $f(x)$ at $x = 0$ is
$Q(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2}(x-0)^2$. Now
$f'(0) = \cos(0) = 1$. $f'(0) = -2\sin(0) = 0$,
$f''(0) = -4\cos(0) = -4$. Hence $Q(x) = 1 - 2x^2$.

6. D: $\lim_{x\to -3^-} e^{1/(x+3)} = e^{\lim_{x\to -3^-} 1/(x+3)} = e^{-\infty} = 0$

7. B: $x = t^2 + 5$, $y = \sqrt{t}$. The parameter $t = 4$ corresponds to the point $(21, 2)$. Thus $m = \frac{dy/dt}{dx/dt}$ evaluated at $t = 4$.
$m = \frac{\frac{1}{2\sqrt{t}}}{2t} = \frac{1}{2\sqrt{t}(2t)}$. Thus when $t = 4$, $m = \frac{1}{32}$.

8. B: $s(t) = \cos t + \frac{1}{4}t^2$. We want to solve $a(t) = 0$, where $a(t)$ is the acceleration. Now,
$s(t) = \cos t + \frac{1}{4}t^2$, thus $v(t) = -\sin t + \frac{1}{2}t$, so
$a(t) = -\cos t + \frac{1}{2}$. Now, $a(t) = 0$ if $\cos t = \frac{1}{2}$, and this is true if $t = \frac{\pi}{3}$ or $t = \frac{5\pi}{3}$.

9. E: $x^3 + y^3 = 6xy$. Differentiating implicitly,
$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$. Substitute $x = 3$, $y = 3$ and solve for $\frac{dy}{dx}$. $3(9) + 3(9)\frac{dy}{dx} = 6(3) + 6(3)\frac{dy}{dx}$.
$27 + 27\frac{dy}{dx} = 18 + 18\frac{dy}{dx}$, thus $9 = -9\frac{dy}{dx}$, hence $\frac{dy}{dx} = -1$

10. E: $s(t) = t^2 - 2t + 3$. To find the total distance traveled in the first 3 seconds, we must first find where the velocity is positive and negative. $v(t) = 2t - 2$. Thus the particle changes direction at $t = 1$, and more specifically, the particle is moving in the negative direction between $t = 0$ and $t = 1$ and the particle is moving in the positive direction between $t = 1$ and $t = 3$. Hence the total distance traveled in the first 3 seconds is $|s(1) - s(0)| + s(3) - s(1)$
$= |-1| + 4 = 5$ feet.

11. B: The tangent line is vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. Now $x = t^2 - 2t + 4$ and $y = t^3 - 3t^2$. $\frac{dx}{dt} = 2t - 2$. $\frac{dx}{dt} = 0$ if $t = 1$. Note that $\frac{dy}{dt} = 3t^2 - 6t \neq 0$ at $t = 1$. Now if $t = 1$, $x = 3$ and $y = -2$. Hence the point is $(3, -2)$. .

12. (i) $f(x) = x\cos^4(x^3)$. By the product and chain rule,

$f'(x) = (x)\left(4\cos^3(x^3)(-\sin(x^3))(3x^2)\right) + \cos^4(x^3)$

$= -12x^3\cos^3(x^3)\sin(x^3) + \cos^4(x^3)$.

(ii) $g(t) = \sqrt[3]{6t - t^2} = (6t - t^2)^{1/3}$. By the chain rule,

$g'(t) = \frac{1}{3}(6t - t^2)^{-2/3}(6 - 2t)$

$= \frac{6 - 2t}{3(6t - t^2)^{2/3}}$

(iii) $h(x) = e^{\sec\sqrt{x}}$, thus by the chain rule,

$h'(x) = e^{\sec\sqrt{x}}\sec\sqrt{x}\tan\sqrt{x}(\frac{1}{2\sqrt{x}})$

13. Let A be the area, h be the height, and b be the base of the triangle at time t.

We are given $\frac{dh}{dt} = -\frac{1}{2}$cm/min $\frac{dA}{dt} = 2$ cm^2/min. We want to find $\frac{db}{dt}$ when $h = 6$ cm

and $A = 30$ cm^2. The Area of a triangle is $A = \frac{1}{2}bh$. Differentiate implicitly with respect to time using the product rule:

$\frac{dA}{dt} = \frac{1}{2}\frac{db}{dt}h + \frac{1}{2}b\frac{dh}{dt}$. Now, when $h = 6$ cm and $A = 30$ cm^2, $b = 10$ cm. Hence if we substitute $\frac{dh}{dt} = -\frac{1}{2}$cm/min, $\frac{dA}{dt} = 2$ cm^2/min, $h = 6$ cm, $A = 30$ cm^2 and $b = 10$ cm, we obtain

$2 = \frac{1}{2}\frac{db}{dt}(6) + \frac{1}{2}(10)(-\frac{1}{2})$. Thus $\frac{db}{dt} = \frac{3}{2}$ cm/min.

14. To find $\frac{dy}{dx}$, or equivalently, y', differentiate

$\sin(5y + 7x) = 4x^2 + y^3$ implicitly with respect to x using the product rule and chain rule:

$\cos(5y + 7x)(5y' + 7) = 8x + 3y^2y'$

$5\cos(5y + 7x)y' + 7\cos(5y + 7x) = 8x + 3y^2y'$

$y'(5\cos(5y + 7x) - 3y^2) = 8x - 7\cos(5y + 7x)$

$y' = \frac{8x - 7\cos(5y + 7x)}{5\cos(5y + 7x) - 3y^2}$.

15. Recall if g is the inverse of f, then

$g'(a) = \frac{1}{f'(g(a))}$. Thus $g'(2) = \frac{1}{f'(g(2))}$.

Since $f(0) = 2$, $g(2) = 0$. $g'(2) = \frac{1}{f'(0)}$. Now $f'(x) = 3e^{3x} + 6$, hence $f'(0) = 9$. Therefore $g'(2) = \frac{1}{f'(0)} = \frac{1}{9}$

16. $y = \frac{3x + 1}{5 - x}$. To find $f^{-1}(x)$, first interchange x and y:

$x = \frac{3y + 1}{5 - y}$

$x(5 - y) = 3y + 1$

$5x - xy = 3y + 1$

$5x - 1 = y(3 + x)$, thus $f^{-1}(x) = \frac{5x - 1}{3 + x}$

17. (a) The linear approximation for $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$. Here, $a = 4$. hence $L(x) = f(4) + f'(4)(x - 4)$. Now $f(x) = \sqrt{x}$, thus $f(4) = 2$. $f'(x) = \frac{1}{2\sqrt{x}}$, thus $f'(4) = \frac{1}{4}$.

$L(x) = 2 + \frac{1}{4}(x - 4)$, or $L(x) = 1 + \frac{1}{4}x$

(b) Now, $f(x) \approx L(x)$ for x near a. Thus

$\sqrt{x} \approx 1 + \frac{1}{4}x$ for x near 4.

Thus $\sqrt{4.1} \approx 1 + \frac{1}{4}(4.1) = \frac{81}{40} = 2.025$