Math 151 Spring 2004 Exam I Solutions

1. c: By the product rule,

   $h'(x) = f(x)g'(x) + f'(x)g(x)$. Therefore $h'(2) = f(2)g'(2) + f'(2)g(2)$. Plugging in the given information, $h'(2) = \frac{1}{2}(2) + (-3)(5) = -14$.

2. d: In order for $f(x)$ to be continuous at $x = 3$, we must have $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(3)$. This gives $3 - c = 3c - 3 \Rightarrow \frac{3}{2} = c$.

3. b: $y' = 4x^3 + 3x^2 - 5$, so the slope of the tangent line at $x = 2$ is $m = y'(2) = 4(8) + 3(4) - 5 = 39$.

4. e: Let $\mathbf{a} = \langle -3, 4\rangle$ and $\mathbf{b} = \langle 1, 3\rangle$. Then

   $\mathbf{proj_a b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \frac{\langle -3,4\rangle\cdot\langle 1,3\rangle}{|\langle -3,4\rangle|^2}\langle -3,4\rangle$

   $= \frac{-3+12}{(\sqrt{25})^2}\langle -3,4\rangle = \left\langle \frac{-27}{25}, \frac{36}{25}\right\rangle$

5. e: We must choose the interval that contains a solution to the equation $f(x) = 12$. Note $f(2) = 1 < 12$ and $f(3) = 14 > 12$. Since f, a polynomial, is continuous everywhere, there is a solution to the equation $f(x) = 12$ on the interval $[2, 3]$ by the Intermediate Value Theorem.

6. b: $\lim_{x\to\pm\infty} \frac{3x^2 - 2x - 1}{x^2 - 4} = \lim_{x\to\pm\infty} \frac{3 - 2/x - 1/x^2}{1 - 4/x^2} = 3$, thus $y = 3$ is a horizontal asymptote.

7. b: Factor: $\lim_{x\to 1} \frac{(x+1)(x-1)}{(x-1)(x-2)} =$

   $= \lim_{x\to 1} \frac{x+1}{x-2} = -2$

8. a: Let $\mathbf{r(t)} = \langle t^2 + t + 1, 7t - 9\rangle$. Then

   $\mathbf{r'(t)} = \langle 2t + 1, 7\rangle$, so $\mathbf{r'(3)} = \langle 7, 7\rangle$.

9. a: A vector equation of a line passing through the point r_0 and parallel to the vector $\vec{v}$ is $\vec{r_0} + t\vec{v}$. If we choose $r_0 = (-1, 1)$ and $\vec{v} = \vec{AB}$, then

   $\vec{r_0} + \vec{AB} = \langle -1, 1\rangle + t\langle 3, 2\rangle = \langle -1 + 3t, 1 + 2t\rangle$

10. e: First recall that if $\lim_{x\to a} |f(x)| = 0$, then $\lim_{x\to a} f(x) = 0$. Therefore if I show that $\lim_{x\to 0} \left|(\sin x)(\sin\frac{1}{x})\right| = 0$, then it will follow that $\lim_{x\to 0}(\sin x)(\sin\frac{1}{x}) = 0$.

    I will apply the sqeeze theorem.

    Note that $0 \le \left|(\sin x)(\sin\frac{1}{x})\right| \le \sin x$. Also, $\lim_{x\to 0} 0 = 0$ and $\lim_{x\to 0}\sin x = 0$, hence by the squeeze theorem, $\lim_{x\to 0}\left|(\sin x)(\sin\frac{1}{x})\right| = 0$, and therefore $\lim_{x\to 0}(\sin x)(\sin\frac{1}{x}) = 0$.

11. b: First, I will find the slope of the line $3x + 2y = 1$ by solving for y: $y = -\frac{3}{2}x + \frac{1}{2}$. Thus the slope of the line is $-\frac{3}{2}$. Now, the vector $v = \langle -2, 3\rangle$ has a slope of $-\frac{3}{2}$, thus is parallel to the line.

12. First we will find the slope of the tangent line by evaluating $y'\left(\frac{1}{2}\right)$. $y' = 2x - \frac{1}{x^2} \Rightarrow y'\left(\frac{1}{2}\right) = -3$. Since $y(1/2) = 9/4$,

    an equation of the tangent line is

    $y - \frac{9}{4} = -3(x - \frac{1}{2})$.

13. a.) Quotient rule: $f'(x) = \frac{(6x-2)(x^2-4)}{(x^2-4)^2}$

    $= \frac{2x^2 - 22x + 8}{(x^2-4)^2}$

    b.) Product rule: $f'(x) = (x^3 - x + 1)(8x^3 - 14x) + (3x^2 - 1)(2x^4 - 7x^2 - 3)$

    $= 14x^6 - 45x^4 + 8x^3 + 12x^2 - 14x + 3$

14. Let $A = (1, 1)$, $B = (2, -3)$ and $C = (-4, -1)$. We are looking for angle B.

    $\cos\beta = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}||\vec{BC}|} = \frac{\langle -1,4\rangle\cdot\langle -6,2\rangle}{|\langle -1,4\rangle||\langle -6,2\rangle|} = \frac{14}{\sqrt{17}\sqrt{40}}$. This gives $\beta = \arccos(\frac{14}{\sqrt{17}\sqrt{40}}) \approx 58°$.

15. a) $\lim_{x\to 2}\frac{x^3+1}{x^2+1} = \frac{9}{5}$.

    b) $\lim_{x\to 2}\frac{x^3 - 2x^2 + 4x - 8}{x^2 - 4}$

    $= \lim_{x\to 2}\frac{x^2(x-2) + 4(x-2)}{(x+2)(x-2)}$

    $= \lim_{x\to 2}\frac{(x-2)(x^2+4)}{(x-2)(x+2)} = 2$.

16. $C(x) = x^3 + 5x^2$, thus $C'(x) = 3x^2 + 10x$. Thus the rate of increase of $C(x)$ at $x = 3$ is $C'(3) = \$57$.

17. $$f'(9) = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9}$$
$$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$
$$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$
$$= \lim_{x \to 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)} = \frac{1}{6}$$