Math 151 Spring 2004 Exam II Solutions

1. d: By the chain rule, if
$f(x) = (x^3 + 3x - 1)^{\frac{1}{2}}$, then
$f'(x) = \frac{1}{2}\left(x^3 + 3x - 1\right)^{\frac{-1}{2}}(3x^2 + 3) = \frac{3x^2 + 3}{2\sqrt{x^3 + 3x - 1}}$.

2. e: If $y = f(u)$ and $u = g(x)$, then $y = f(g(x))$. We will use the chain rule to find $\frac{dy}{dx}$.
$\frac{dy}{dx} = f'(g(x))g'(x) \Rightarrow \frac{dy}{dx}|_{x=2} = f'(g(2))g'(2) = f'(7)(5) = 20$.

3. c: $\mathbf{r}(t) = \langle t^3 - t, 2t^2 + 3t\rangle$, thus
$\mathbf{v}(t) = \langle 3t^2 - 1, 4t + 3\rangle$, hence $\mathbf{a}(t) = \langle 6t, 4\rangle$. Now $\mathbf{a}(2) = \langle 12, 4\rangle$.

4. e: Let $y = \frac{3x - 5}{7x + 2}$. Interchange x and y:
$x = \frac{3y - 5}{7y + 2}$. Now solve for y: $x(7y + 2) = 3y - 5 \Rightarrow 7xy + 2x = 3y - 5$, thus $2x + 5 = y(3 - 7x) \Rightarrow y = \frac{2x + 5}{3 - 7x}$.

5. b: Recall $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Thus $\frac{dy}{dx} = \frac{9t^2 - 1}{2t + 3}$, now replace t with 2, we will find $m = \frac{35}{7} = 5$.

6. e: $e^{3\ln\sqrt{5} - 2\ln 3 + 7}$
$= \frac{e^{3\ln\sqrt{5} + 7}}{e^{2\ln 3}}$
$= \frac{e^{\ln 5^{3/2}}e^7}{e^{\ln 9}}$
$= \frac{5^{3/2}e^7}{9} = \frac{5\sqrt{5}e^7}{9}$

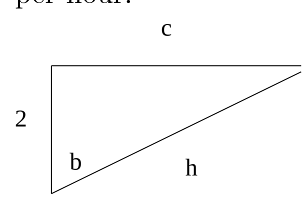
7. a: $\lim_{x\to 0}\frac{\tan x - \sin x}{x^2}$
$= \lim_{x\to 0}\frac{\frac{\sin x}{\cos x} - \sin x}{x^2}$
$= \lim_{x\to 0}\frac{\sin x - \sin x\cos x}{x^2}$
$= \lim_{x\to 0}\frac{\sin x(1 - \cos x)}{x^2}$
$= \lim_{x\to 0}\frac{\sin x}{x}\frac{1 - \cos x}{x} = (1)(0) = 0$.

8. b: $f(x) = e^{3x} - 7e^{-2x} + 5e^x - 3e^{-x}$
$\Rightarrow f'(x) = 3e^{3x} + 14e^{-2x} + 5e^x - 3e^{-x}$.

9. a: $L(x) = f(81) + f'(81)(x - 81)$. $f(x) = \sqrt{x} \Rightarrow f(81) = 9$ and $f'(81) = \frac{1}{18}$. Therefore
$L(x) = 9 + \frac{1}{18}(x - 81)$, thus $\sqrt{79} \approx L(79)$ where
$L(79) = 9 + \frac{1}{18}(-2) = \frac{80}{9}$.

10. e: Let $\mathbf{r}(t) = \langle\sin 3t, t + \cos 2t\rangle$. Then
$\mathbf{r}'(t) = \langle 3\cos 3t, 1 - 2\sin 2t\rangle$. $\mathbf{r}'(0) = \langle 3, 1\rangle$. Thus the speed at $t = 0$ is $|\mathbf{r}'(0)| = \sqrt{10}$.

11. Refer to the figure below. c is the path of the car, and b is the angle of the camera. We were given that $\frac{dc}{dt} = 60$. We wish to find $\frac{db}{dt}$ when $c = 3$. $\tan b = \frac{c}{2}$, thus $\sec^2 b\frac{db}{dt} = \frac{1}{2}\frac{dc}{dt}$, thus since we know $\frac{dc}{dt} = 60$ and since h is $\sqrt{13}$ when $c = 3$, we also know $\sec b = hyp/adj = \frac{\sqrt{13}}{2}$. Plugging all of this information in to the expression $\sec^2 b\frac{db}{dt} = \frac{1}{2}\frac{dc}{dt}$, we find $\left(\frac{\sqrt{13}}{2}\right)^2\frac{db}{dt} = 30$. Thus $\frac{db}{dt} = \frac{120}{13}$ radians per hour.



12. a.) Let $f(x) = x^3 - 5$, then the solution to the equation $f(x) = 0$ is $x = \sqrt[3]{5}$.
b.) $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. Here, $x_1 = 2$. Hence
$x_2 = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{3}{12} = \frac{7}{4}$
$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$ where $x_2 = 7/4$. Hence
$x_2 = 7/4 - \frac{f(7/4)}{f'(7/4)} = \frac{503}{294}$

13. Using logarithm properties,

$\ln x + \ln(x-3) = \ln 5 + 3\ln 2$ is equivalent to

$\ln x(x-3) = \ln 40$. This gives

$x(x-3) = 40 \Rightarrow x^2 - 3x - 40 = 0 \Rightarrow$

$(x-8)(x+5) = 0$, giving $x = 8$ or $x = -5$. Now $x = -5$ is an extraneous root since $\ln(-5)$ is not defined. Thus the only solution is $x = 8$.

14. a.) $f(x) = (x^3 + x + 2)^{17}$. By the chain rule,

$f'(x) = 17(x^3 + x + 2)^{16}(3x^2 + 1)$.

b.) $f(x) = e^{1+x+\sqrt[3]{x}}$, so by the chain rule,

$$f'(x) = \left(1 + \frac{1}{3}x^{-\frac{2}{3}}\right) e^{1+x+\sqrt[3]{x}}$$

c.) $f(x) = \sin^3 x + \tan^2 5x$, then

$f'(x) = 3\sin^2 x \cos x + 2\tan(5x)\sec^2(5x)(5)$.

15. a.) $\displaystyle\lim_{x\to 0} \frac{\tan 3x \sin 5x}{2x^2}$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\tan 3x \sin 5x}{x^2}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\tan 3x}{x}\frac{\sin 5x}{x}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\sin 3x}{x\cos 3x}\frac{\sin 5x}{x}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{1}{\cos 3x}\frac{\sin 3x}{x}\frac{\sin 5x}{x}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{1}{\cos 3x}\frac{3\sin 3x}{3x}\frac{5\sin 5x}{5x}$$

$$= \frac{15}{2}\lim_{x\to 0} \frac{1}{\cos 3x}\frac{\sin 3x}{3x}\frac{\sin 5x}{5x}$$

$$= \frac{15}{2}(1)(1) = \frac{15}{2}$$

b.) Recall $\cos t = \sin(\frac{\pi}{2} - t)$.

Thus $\cos\frac{x}{2} = \sin(\frac{\pi}{2} - \frac{x}{2})$

$= \sin(\frac{1}{2}(\pi - x)) = -\sin(\frac{1}{2}(x - \pi))$.

Now applying this to the limit,

$$\lim_{x\to\pi} \frac{\cos(\frac{1}{2}x)}{x - \pi}$$

$$= \lim_{x\to\pi} \frac{-\sin(\frac{1}{2}(x-\pi))}{x - \pi}$$

$$= -\frac{1}{2}\lim_{x\to\pi} \frac{\sin(\frac{1}{2}(x-\pi))}{\frac{1}{2}(x-\pi)} = -\frac{1}{2}(1) = -\frac{1}{2}$$

16. Differentiate implicitly.

$2xy^2 + 2x^2yy' + 15x^2y + 5x^3y' + y^3 + 3xy^2y' = 0$.

$y'(2x^2y + 5x^3 + 3xy^2) = -2xy^2 - 15x^2y - y^3$.

$y' = \dfrac{-2xy^2 - 15x^2y - y^3}{2x^2y + 5x^3 + 3xy^2}$. Now plug in the point $(1,1)$, we get

$m = \dfrac{-18}{10} = -\dfrac{9}{5}$. Thus the equation of the tangent line is $y - 1 = -\dfrac{9}{5}(x-1)$

17. $Q(x) = f(8) + f'(8)(x-8) + \dfrac{f''(8)}{2}(x-8)^2$. Now $f(x) = \sqrt[3]{x}$, so $f(8) = 2$, $f'(8) = \dfrac{1}{12}$ and $f''(8) = -\dfrac{1}{144}$.

Thus $Q(x) = 2 + \dfrac{1}{12}(x-8) - \dfrac{1}{288}(x-8)^2$.