Math 151 Spring 2005 Exam I Solutions

1. E: $\langle 3,5\rangle \cdot (\langle 3,5\rangle + \langle -4,2\rangle) = \langle 3,5\rangle \cdot \langle -1,7\rangle = 32$

2. B: $\cos\phi = \dfrac{\vec{PQ}\cdot\vec{PR}}{|\vec{PQ}||\vec{PR}|} = \dfrac{\langle 2,-2\rangle\cdot\langle 3,-4\rangle}{|\langle 2,-2\rangle||\langle 3,-4\rangle|} = \dfrac{7}{5\sqrt{2}}$

3. C: $\langle -4,7\rangle = x\langle 1,2\rangle + y\langle -1,3\rangle$

   $\Rightarrow \langle -4,7\rangle = \langle x-y, 2x+3y\rangle$. In order to find the solution to this equation, we need to equate components:

   $x-y=-4$ and $2x+3y=7$, yielding $x=-1$.

4. C: Recall if $\mathbf{u}$ and $\mathbf{v}$ are vectors,

   then $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$. Now, since $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ form an equilateral triangle (all are unit vectors), $\theta = 60°$. Hence

   $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos 60° = (1)(1)(\frac{1}{2}) = \frac{1}{2}$.

5. C: $\dfrac{f(2+h)-f(2)}{h}$

   $= \dfrac{(2+h)^2+3(2+h)+1-(4+6+1)}{h}$

   $= \dfrac{h^2+7h}{h} = h+7$.

6. D: $y=f(x)=x^{1/2}$, thus $f'(x) = 1/2x^{-1/2} = \dfrac{1}{2\sqrt{x}}$.

   Now, $m = f'(4) = \dfrac{1}{4}$. An equation of the tangent line is $y-y_1 = m(x-x_1)$ where $x_1=4$ and

   $y_1 = f(4) = \dfrac{1}{2}$. This gives $y - \dfrac{1}{2} = \dfrac{1}{4}(x-4) \Rightarrow$ $y = \dfrac{x}{4}+1$.

7. A: $\displaystyle\lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)(x+1)} = \lim_{x\to 2}\frac{x+2}{x+1} = \frac{4}{3}$

8. D: $\displaystyle\lim_{x\to 2^-}\frac{|x-2|}{|x^2-4|} = \lim_{x\to 2^-}\frac{-(x-2)}{(x+2)(x-2)}$

   $\displaystyle= \lim_{x\to 2^-}\frac{-1}{(x+2)} = \frac{-1}{4}$

9. C: $f(x)$ is not differentiable at $x=1$ because of a discontinuity. $f(x)$ is not differentiable at $x=3$ because the graph of $f(x)$ has a cusp (sharp point) there.

10. E: Conjugate the numerator:

    $\displaystyle\lim_{x\to\infty}\frac{(\sqrt{x^2+3x+3}-\sqrt{x^2+4})(\sqrt{x^2+3x+3}+\sqrt{x^2+4})}{\sqrt{x^2+3x+3}+\sqrt{x^2+4}}$

    $\displaystyle= \lim_{x\to\infty}\frac{3x-1}{\sqrt{x^2+3x+3}+\sqrt{x^2+4}}$. Now divide the numerator and denominator by x:

    $\displaystyle= \lim_{x\to\infty}\frac{3-1/x}{\sqrt{1+3/x+3/x^2}+\sqrt{1+4/x^2}} = \frac{3}{2}$.

11. B: Let $f(x) = x^3-3x+1$. Since $f(x)$ is a continuous function, we will apply the Intermediate Value Theorem on the interval $(1,2)$. Note that $f(1) = -1 < 2.5$ and $f(2) = 3 > 2.5$. Thus by IVT, there must be a solution to $f(x) = 2.5$ on the interval $(1,2)$. Hence $k=2.5$.

12. A: $\displaystyle\lim_{x\to\pm\infty}\frac{2x+1}{3x^2+x-4} = \lim_{x\to\pm\infty}\frac{2/x+1/x^2}{3+1/x-4/x^2} = 0$. Thus there is a horizontal asymptote at $y=0$.

13. C: We will apply the product rule to the function $h = f(2f+g)$. $h' = f(2f'+g') + f'(2f+g)$. Thus $h'(1) = f(1)(2f'(1)+g'(1)) + f'(1)(2f(1)+g(1))$. Plugging in the given information, we then have $h'(1) = 2(2*2-5)+2(2*3+3) = 15$.

14. a.) The vector equation of the line is given by $L(t) = \vec{r_0} + t\vec{v}$, where r_0 is any point on the line and $\vec{v}$ is parallel to the line. We can choose r_0 to be the point A or B (we will choose A) and $\vec{v}$ is either $\vec{AB}$ or $\vec{BA}$ (we will choose $\vec{AB}$). Therefore $L(t) = \vec{r_0} + t\vec{v} = \langle 2,-1\rangle + t\langle 1,6\rangle$, hence $L(t) = \langle 2+t, -1+6t\rangle$.

    b.) Since $\vec{v} = \vec{AB} = \langle 1,6\rangle$ is parallel to the line, $\mathbf{n} = \vec{v}^{\perp} = \langle -6,1\rangle$ is perpendicular to the line.

    c.) $comp_{\mathbf{n}}(\vec{CA}) = \dfrac{\mathbf{n}\cdot\vec{CA}}{|\mathbf{n}|}$

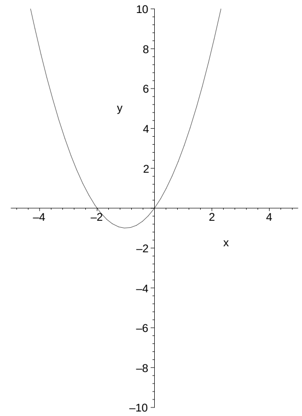
    $= \dfrac{\langle -6,1\rangle\cdot\langle 5,-3\rangle}{|\langle -6,1\rangle|} = \dfrac{-33}{\sqrt{37}}$

    d.) $d = \left|\dfrac{-33}{\sqrt{37}}\right| = \dfrac{33}{\sqrt{37}}$.

15. a.) $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$.

b.) $\lim_{x \to 4} \frac{f(x) - 3}{x - 4} = \lim_{x \to 4} \frac{f(x) - f(4)}{x - 4}$

(since $3 = f(4)$). Now, $\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = f'(4)$, by the definition of the derivative. Since it is given that $f'(x) = \frac{1}{\sqrt{2x+1}}$, $f'(4) = \frac{1}{\sqrt{9}} = \frac{1}{3}$.



16. $f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$

$$= \lim_{x \to 2} \frac{\frac{x}{x+1} - \frac{2}{3}}{x - 2}$$

$$= \lim_{x \to 2} \frac{3x - 2(x+1)}{(x-2)(x+1)(3)}$$

$$= \lim_{x \to 2} \frac{x - 2}{(x-2)(x+1)(3)} = \frac{1}{9}$$

17. Using the Quotient Rule,

$$f'(x) = \frac{(2)(x^2 + 3x - 1) - (2x - 5)(2x + 3)}{(x^2 + 3x - 1)^2}$$

$$= \frac{-2x^2 + 10x + 13}{(x^2 + 3x - 1)^2}.$$

18. First note that each piece of this piece-wise function are polynomials, hence continuous. Thus the only place we must force continuity is where the domain splits, namely at $x = 1$. Now,in order for $f(x)$ to be continuous at $x = 1$, we must have

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = f(1)$. Since $f(1) = 3$, We need to solve the following equations:

$\lim_{x \to 1^+} f(x) = 3$ and $\lim_{x \to 1^-} f(x) = 3$.

$1 + a + b = 3$ and $-a + 2b + 6 = 3$. Adding these two equations yields $3b + 7 = 6 \Rightarrow b = -\frac{1}{3} \Rightarrow a = \frac{7}{3}$.

19. a.) $x = t - 1$, $y = t^2 - 1$.

b.) $x = t - 1 \Rightarrow t = x + 1$. This gives $y = (x+1)^2 - 1$.

c.) The direction of the curve moves from left to right.