Math 151 Spring 2007 Exam II Solutions

1. E: $\lim_{x\to 0} \dfrac{2\cos x}{\sin(2x)(x+1)} = \dfrac{2(1)}{0(1)}$, hence the limit does not exist.

2. B: Let $y = \dfrac{2x-3}{7x+5}$. Interchange x and y:

   $x = \dfrac{2y-3}{7y+5}$. Now we will solve for y:

   $x(7y+5) = 2y-3 \Rightarrow 7xy + 5x = 2y - 3$

   $\Rightarrow 5x + 3 = 2y - 7xy \Rightarrow \dfrac{5x+3}{2-7x}$.

   Thus $f^{-1}(x) = \dfrac{5x+3}{2-7x}$

3. A: $f(x) = \dfrac{1+\cos x}{\sin x}$. Using the quotient rule,

   $f'(x) = \dfrac{-\sin x(\sin x) - (1+\cos x)(\cos x)}{\sin^2 x}$

   $= \dfrac{-(\sin^2 x + \cos^2 x) - \cos x}{\sin^2 x}$

   $= \dfrac{-1-\cos x}{\sin^2 x}$.

4. C: $\lim_{x\to 0} \dfrac{e^{\sin x}-1}{x}$ is of the form $\dfrac{0}{0}$, so we will use L'Hospitals Rule.

   $\lim_{x\to 0} \dfrac{e^{\sin x}-1}{x} \stackrel{L}{=} \lim_{x\to 0} \dfrac{\cos x e^{\sin x}}{1} = 1$.

5. B: Using the Chain rule, if $h(x) = f(g(x))$,

   $h'(x) = f'(g(x))g'(x)$.

   Thus $h'(1) = f'(g(1))g'(1) = f'(4)(2) = 2(2) = 4$.

6. E: $f(x) = x^e + e^x \Rightarrow f'(x) = ex^{e-1} + e^x$, thus

   $f'(1) = ex^0 + e^1 = 2e$.

7. D: We will differentiate $x^4 + y^2 = 25$ implicitly and plug in the point $(2,3)$ to get the slope of the tangent line.

   $4x^3 + 2yy' = 0 \Rightarrow y' = \dfrac{-2x^3}{y}$. Hence the slope of the tangent line is $m = \dfrac{-2(8)}{3} = -\dfrac{16}{3}$. Thus an equation of the tangent line is

   $y - 3 = -\dfrac{16}{3}(x-2)$.

8. A: $\langle x, y\rangle = \langle \cos t + \sin t, \cos^2 t + 1\rangle$

   $\Rightarrow \langle x, y\rangle' = \langle -\sin t + \cos t, -2\sin t\cos t\rangle$. Note that the point $(2,3)$ corresponds to $t = 0$.

   Thus if we evaluate $\langle x, y\rangle'$ at $t = 0$, we obtain $\langle 1, 0\rangle$. Thus the slope of the tangent line is

   $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{0}{1} = 0$.

9. D: Let $Q(x) = ax^2 + bx + c$. Then $Q'(x) = 2ax + b$ and $Q''(x) = 2a$. Solving $Q''(0) = 2$ gives us $2a = 2$ and thus $a = 1$. Now $Q'(x) = 2x + b$. Solving $Q'(0) = -1$ gives $b = -1$. Now $Q(x) = x^2 - x + c$, and solving $Q(1) = 3$ gives $1 - 1 + c = 3 \Rightarrow c = 3$. Hence $Q(x) = x^2 - x + 3 \Rightarrow Q(2) = 5$.

10. D: The differential is defined as $dy = f'(x)dx$. Now, $f(x) = \sin(\cos x)$. By the chain rule,

    $f'(x) = \cos(\cos x)(-\sin x) = -\cos(\cos x)(\sin x)$, thus

    $dy = \cos(\cos x)(-\sin x)dx = -\cos(\cos x)(\sin x)dx$

11. C: Let $f(x) = \cos x$. The quadratic approximation at $x = 0$ is

    $Q(x) = f(0) + f'(0)(x-0) + \dfrac{f''(0)}{2}(x-0)^2$.

    Now, $f(0) = \cos(0) = 1$, $f'(0) = -\sin(0) = 0$ and $f''(0) = -\cos(0) = -1$. Plugging in this information gives $Q(x) = 1 - \dfrac{1}{2}x^2$, thus

    $\cos(\frac{1}{3}) \approx Q(\frac{1}{3}) = 1 - \dfrac{1}{18} = \dfrac{17}{18}$.

12. E: Since we are given that $g = f^{-1}$ and $f(1) = 2$, then $g(2) = 1$. Using the formula $g'(2) = \dfrac{1}{f'(g(2))}$,

    $g'(2) = \dfrac{1}{f'(1)} = \dfrac{1}{3}$

13. (a) Newton's Method tells us that if we use x_1 as an initial guess of the root of $f(x)$, then x_2, the second approximation, is given by $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$ provided $f'(x_1) \neq 0$. Now we will let $f(x) = x^2 - 3$ and we are given that $x_1 = 2$.

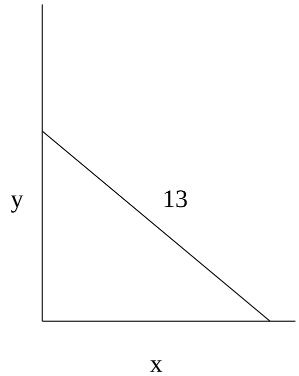
    $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{1}{4} = \dfrac{7}{4}$

    (b) $x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = \dfrac{7}{4} - \dfrac{f(7/4)}{f'(7/4)} = \dfrac{7}{4} - \dfrac{1/16}{7/2} = \dfrac{97}{56}$

14. Refer to the figure below. We are given $\frac{dx}{dt} = 7$. We want to find $\frac{dy}{dt}$ when $x = 5$. By Pythagorean Theorem, $x^2 + y^2 = 169$. Differentiate implicitly with respect to t gives us

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. Thus $\frac{dy}{dt} = \frac{-x\,dx/dt}{y}$. When $x = 5$, $y = \sqrt{169 - x^2} = \sqrt{144} = 12$.

$\frac{dx}{dt} = \frac{-5(7)}{12} = -\frac{35}{12}$ feet per second.



15. $f(x) = (3x - 2)^{1/2}$.

$f'(x) = \frac{1}{2}(3x-2)^{-1/2}(3) = \frac{3}{2}(3x-2)^{-1/2}$

$f''(x) = -\frac{3}{4}(3x-2)^{-3/2}(3) = -\frac{9}{4}(3x-2)^{-3/2}$

16. (a) $e^{x+y} = x^2 + y$

$e^{x+y}(1 + \frac{dy}{dx}) = 2x + \frac{dy}{dx}$

$e^{x+y} + e^{x+y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$

$e^{x+y} - 2x = \frac{dy}{dx} - e^{x+y}\frac{dy}{dx}$

Thus $\frac{dy}{dx} = \frac{e^{x+y} - 2x}{1 - e^{x+y}}$

(b) $3x^2 + y^3 + xy = 0$

$6x + 3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 + x) = -y - 6x$

$\frac{dy}{dx} = \frac{-y-6x}{3y^2+x}$

17. $f(x) = \tan(x^2 - x + 1)$. By the chain rule,

$f'(x) = \sec^2(x^2 - x + 1)(2x - 1)$.

18. (a) $\vec{r}(t) = \langle 2\cos t, 2\sin t\rangle$. Thus the velocity is given by

$\vec{v}(t) = \langle -2\sin t, 2\cos t\rangle$.

(b) The acceleration is $\vec{a}(t) = \langle -2\cos t, -2\sin t\rangle$.

(c) Two vectors are perpendicular if their dot product is zero.

$\vec{v}(t)\cdot\vec{a}(t) = \langle -2\sin t, 2\cos t\rangle \cdot \langle -2\cos t, -2\sin t\rangle = 4\sin t\cos t - 4\cos t\sin t = 0$.