Math 151 Spring 2007 Exam III Solutions

1. B: We can see from looking at choice (b) that $f(x)$ has a local maximum and $f(x)$ changes from concave down to concave up at this local maximum, hence is also a point of inflection.

2. D: $f(x) = 3x^4 - 8x^3 + 5$, thus $f'(x) = 12x^3 - 24x^2$ $= 12x^2(x-2)$. This gives critical values of $x = 0$ and $x = 2$. To determine whether these critical values yield a minima, we will construct the first derivative sign chart:

f'	$-$	|	$-$	|	$+$
f	$\searrow$	|	$\searrow$	|	$\nearrow$
x		0		2	

We see from viewing the sign of the first derivative, $f(x)$ has a local minimum at $x = 2$ and no extrema at $x = 0$.

3. B: $f(x) = \tan^{-1}(\ln x)$, By the chain rule,
$$f'(x) = \frac{1}{1+(\ln x)^2}\frac{d}{dx}(\ln x).$$
Thus $f'(x) = \dfrac{1}{1+(\ln x)^2}\dfrac{1}{x}$.

4. $f(x) = 10x^6 - 24x^5 + 15x^4$, thus
$f'(x) = 60x^5 - 120x^4 + 60x^3$ and
$f''(x) = 300x^4 - 480x^3 + 180x^2$
$= 60x^2(5x-3)(x-1)$. Now to determine the inflection points, we will construct the second derivative chart, plotting the zeros of $f''(x)$, namely $x = 0$, $x = 1$ and $x = 3/5$:

f''	$+$	|	$+$	|	$-$	|	$+$
f	$\cup$	|	$\cup$	|	$\cap$	|	$\cup$
x		0		3/5		1	

Thus since f changes concavity at $x = 3/5$ and $x = 1$, these are the inflection points of f.

5. E: Recall $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$.
Thus $\sum_{i=1}^{50} \dfrac{3i-7}{5} = \sum_{i=1}^{50}\dfrac{3i}{5} - \sum_{i=1}^{50}(\dfrac{7}{5})$
$$= \frac{3}{5}\sum_{i=1}^{50}(i) + \frac{7}{5}\sum_{i=1}^{50}(1)$$
$$= \frac{3}{5}\left(\frac{50(51)}{2}\right) + \frac{7}{5}(50) = 695$$

6. C: $f(x) = 2x^5 - 5x^4 - 10x^3$. To find where $f(x)$ is increasing, we will make the first derivative sign chart:
$f'(x) = 10x^4 - 20x^3 - 30x^2 = 10x^2(x-3)(x+1)$. This yields critical values of $x = 0$, $x = 3$ and $x = -1$.

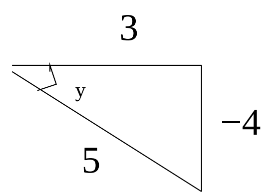
f'	$+$	|	$-$	|	$-$	|	$+$
f	$\nearrow$	|	$\searrow$	|	$\searrow$	|	$\nearrow$
x		-1		0		3	

We see from the sign chart that f is increasing for $x < -1$ or $x > 3$.

7. C: $\lim_{x\to 0} \dfrac{1-\cos^3 x}{\sin^2 x + \sin(2x)}$ is of the form $\dfrac{0}{0}$, so we will use L'Hospital's rule.
$$\lim_{x\to 0}\frac{1-\cos^3 x}{\sin^2 x + \sin(2x)} \stackrel{L}{=}$$
$$\lim_{x\to 0}\frac{3\cos^2 x \sin x}{2\sin x\cos x + 2\cos(2x)} = 0$$

8. B: We will use Logarithmic Differentiation. Let $y = x^{\ln x}$. Then $\ln y = \ln\left(x^{\ln x}\right) = (\ln x)^2$. Thus
$\dfrac{y'}{y} = 2(\ln x)\dfrac{1}{x}$
$\Rightarrow y' = y\dfrac{2\ln x}{x}$, thus $y' = x^{\ln x}\left(\dfrac{2\ln x}{x}\right)$

9. B: Let $y = \sin^{-1}(-\frac{4}{5})$. Then $\sin y = -\frac{4}{5}$.
By viewing the triangle below,



we see that $\cos y = \dfrac{3}{5}$.

10. A: $e^{3\ln 2 - 1}\ln(5e^2) = e^{\ln 8 - 1}\ln(5e^2)$
$= e^{\ln 8}e^{-1}(\ln(5) + \ln(e^2)) = \dfrac{8}{e}(\ln 5 + 2)$

11. C: Recall that $\dfrac{d}{dx}\arcsin(x) = \dfrac{1}{\sqrt{1-x^2}}$, it then follows that the antiderivative of $\dfrac{1}{\sqrt{1-x^2}}$ is $\arcsin(x) = \sin^{-1}(x)$.

12. A: We must choose the graph that is always increasing and whose derivative is consistently decreasing, meaning f is concave down. Graph (a) is the only one that matches this description.

13. (a) Using the exponential growth model, $y(t) = y_0 e^{kt}$ where $y_0 = 7$ and $y(15\ln 7 - 30\ln 2) = 4 \Rightarrow y(\ln(\frac{7^{15}}{2^{30}})) = 4$.

$y(t) = 7e^{kt}$. $y(\ln(\frac{7^{15}}{2^{30}})) = 4$

$$\Rightarrow 4 = 7e^{k\ln(\frac{7^{15}}{2^{30}})}$$

$$\Rightarrow 4 = 7e^{\left(\ln(\frac{7^{15}}{2^{30}})\right)^k}$$

$$4 = 7\left(\frac{7^{15}}{2^{30}}\right)^k$$

$$\frac{4}{7} = \left(\frac{7^{15}}{2^{30}}\right)^k$$

$$\ln\frac{4}{7} = k\ln\frac{7^{15}}{2^{30}}$$

Hence $k = \dfrac{\ln(4/7)}{\ln(7^{15}/2^{30})}$

Now this gives $y(t) = 7e^{\frac{\ln(4/7)}{\ln(7^{15}/2^{30})}t}$.

(b) Now we must solve $y(t) = \frac{1}{2}y_0$.

$$\frac{7}{2} = 7e^{\frac{\ln(4/7)}{\ln(7^{15}/2^{30})}t}$$

$$\frac{1}{2} = e^{\frac{\ln(4/7)}{\ln(7^{15}/2^{30})}t}$$

$$\ln\frac{1}{2} = \frac{\ln(4/7)}{\ln(7^{15}/2^{30})}t$$

$$t = \frac{\ln(1/2)\ln(7^{15}/2^{30})}{\ln(4/7)} \approx 10.4 \text{ years.}$$

14. (a) $\lim_{x\to 0}\dfrac{\tan^{-1}x}{\sin x}$ is of the form $\dfrac{0}{0}$. Thus we will apply L'Hospitals Rule.

$$\lim_{x\to 0}\frac{\tan^{-1}x}{\sin x} \overset{L}{=} \lim_{x\to 0}\frac{1/(1+x^2)}{\cos x} = 1.$$

(b) $\lim_{x\to\infty}\dfrac{\ln x}{x^{1/3}}$ is of the form $\dfrac{\infty}{\infty}$. Thus we will use L'Hospitals Rule:

$$\lim_{x\to\infty}\frac{1/x}{\frac{1}{3}x^{-2/3}} \overset{L}{=} \lim_{x\to\infty}\frac{1}{x}\frac{3x^{2/3}}{1} = \lim_{x\to\infty}\frac{3}{x^{1/3}} = 0.$$

(c) Let $y = (1+2x)^{3/x}$. Then $\ln y = \ln(1+2x)^{3/x}$

$\ln y = \dfrac{3\ln(1+2x)}{x}$. Now,

$\lim_{x\to 0}\ln y = \lim_{x\to 0}\dfrac{3\ln(1+2x)}{x}$. This limit is of the form 0/0 so we will use L'Hospitals Rule.

$$\lim_{x\to 0}\frac{3\ln(1+2x)}{x} = \lim_{x\to 0}\frac{6}{1+2x} = 6,$$

hence $\lim_{x\to 0}(1+2x)^{3/x} = e^6$.

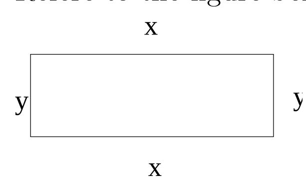
15. $\Delta x = \dfrac{b-a}{n} = \dfrac{4-1}{6} = \dfrac{1}{2}$.

Now the Reimann Sum is

$\frac{1}{2}(f(1) + f(16/9) + f(9/4) + f(25/9) + f(49/16) + f(4))$

$= \frac{1}{2}(1 + 4/3 + 3/2 + 5/3 + 7/4 + 2) = \frac{37}{8}$.

16. Refere to the figure below.



The total cost is given by

$C = \$5x + \$3y + \$3y + \$3x = 8x + 6y$. Using the constraint $xy = 7500 \Rightarrow y = 7500/x$. Thus $C = 8x + 45000/x$. $C' = 8 - 45000/x^2$.

$C' = \dfrac{8x^2 - 45000}{x^2}$, yielding a critical number of $x = 75$. Now $C''(75) > 0$, thus the cost is minimized when $x = 75$ by the second derivative test. Hence the north side must be 75 feet long.

17. (a) Since $a(t) = 30t + 8$, $v(t) = 15t^2 + 8t + v_0$. We were given $v_0 = 44$, thus $v(t) = 15t^2 + 8t + 44$.

(b) $x(t) = 5t^3 + 4t^2 + 44t + s_0$. We were given $s_0 = 100$, hence $x(t) = 5t^3 + 4t^2 + 44t + 100$.

18. Let $y = \dfrac{(2x-1)^3(x+2)^{1/2}}{(5x-1)^{1/3}(3x+5)^2}$. The technique of logarithmic differentiation would be most effective here.

$$\ln y = \ln\frac{(2x-1)^3(x+2)^{1/2}}{(5x-1)^{1/3}(3x+5)^2}$$

$$= \ln\left((2x-1)^3(x+2)^{1/2}\right) - \left(\ln(5x-1)^{1/3}(3x+5)^2\right)$$

$= 3\ln(2x-1) + \frac{1}{2}\ln(x+2) - \frac{1}{3}\ln(5x-1) - 2\ln(3x+5)$.

$\frac{y'}{y} = 3\frac{2}{2x-1} + \frac{1}{2}\frac{1}{x+2} - \frac{1}{3}\frac{5}{5x-1} - 2\frac{3}{3x+5}$.

$y' = y(\frac{6}{2x-1} + \frac{1}{2x+4} - \frac{5}{15x-3} - \frac{6}{3x+5})$, where y is as originally stated.