

Name _____
Section _____

Math 151
Common Exam # 2, Version A
Spring 1998

No calculators are allowed on this exam. The SCANTRONS will be collected at the end of the first hour. Put your name and Section number on your SCANTRON.

Part I.

Each problem is worth 4 points. Mark your answers on your SCANTRON form. Put your name and Section number on your SCANTRON. Indicate on your SCANTRON form whether you have Version A or B of the exam. For your own record, also mark your answers on this exam. The exam will be returned to you; the SCANTRON will not.

1. $\lim_{x \rightarrow \infty} \frac{1 - 2e^{2x}}{2 + 3e^{2x}}$

- a) $-\frac{2}{3}$ b) $\frac{1}{2}$ c) 0 d) does not exist

2. $\lim_{t \rightarrow 0} \frac{\sin^2(3t)}{t^2}$

- a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) 3 d) 9

3. If $f(x) = \tan^2(x) + \tan(x^2)$, $f'(x) =$

- a) $\sec^2 x + \sec^2 x^2$ b) $2 \tan x \sec x + 2x \sec x^2$
c) $2 \tan x \sec^2 x + 2x \sec^2 x^2$ d) $2 \tan x \sec x - 2x \sec x^2$

4. If $f(x) = \frac{\sin^2 x}{\cos x}$, $f'(x) =$

a) $-2 \cos x$ b) $2 + \frac{\sin^2 x}{\cos^2 x}$ c) $2 \sin x - \frac{\sin^3 x}{\cos^2 x}$ d) $2 \sin x + \frac{\sin^3 x}{\cos^2 x}$

5. Suppose $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, $f'(6) = 7$, $f(3) = 2$, and $f(6) = -3$. What is $F'(3)$?

a) 8 b) 28 c) 42 d) 7

6. What is the equation of the line tangent to the graph of $y^2 = x^3(2 - x)$ at the point $(1, 1)$?

a) $y = -x + 2$ b) $y = 2x - 1$ c) $y = 1$ d) $y = x$

7. What is a unit vector tangent to the curve $\mathbf{r}(t) = \langle \cos 2t, \sin 2t \rangle$ when $t = \pi/8$.

a) $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ b) $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$ c) $\langle 0, 1 \rangle$ d) $\langle 1, 0 \rangle$

8. What is the slope of the line tangent to the curve $x = t \ln t$, $y = te^{2t}$ when $t = 1$?

a) $\frac{3}{2}e^2$ b) $\frac{1}{3e^2}$ c) $2e^2$ d) $3e^2$

9. If $y = xe^{-x^2}$, $\frac{dy}{dx} =$

a) $(1 + x)e^{-x^2}$ b) $(1 - 2x^2)e^{-x^2}$ c) $-2x^2e^{-x^2}$ d) $(1 + 2x^2)e^{-x^2}$

10. The slope of the line tangent to the curve $2e^{xy} = x + y$ at the point $(0, 2)$ is

a) 3 b) -1 c) 4 d) 1

11. Find all solutions of $\ln(x + 6) + \ln(x - 3) = \ln 5 + \ln 2$.

- a) -7 and 4 b) 7 and -4 c) 7 d) 4

12. If $f(x) = \ln(x + \ln x)$, what is $f'(e)$?

- a) $\frac{1}{e+1}$ b) $\frac{e}{e+1}$ c) $\frac{1}{e}$ d) $\frac{1}{e^2+e}$

13. If $y = \sin(x)$, what is $\frac{d^{103}y}{dx^{103}}$?

- a) $\sin x$ b) $\cos x$ c) $-\sin x$ d) $-\cos x$

Part 2.

Show all your work.

14. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 cubic centimeter per minute. At what rate is the surface area of the snowball decreasing when the diameter is 10 cm? The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$ and the surface area is given by $S = 4\pi r^2$, where r is the radius of the sphere.

15. If y is implicitly defined as a function of x by the relation $y^5 + 3x^2y^2 + 5x^4 = 12$, find $\frac{dy}{dx}$.

16. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the wall at a speed of 1.6 m/s, how fast is his shadow on the wall decreasing when he is 4 m from the wall?

17. a) Find the inverse of $f(x) = \frac{x-2}{x+2}$.

- b) Let $g(x)$ be a one-to-one function satisfying $g(2) = 3$, $g'(2) = 4$, $g(3) = -1$, $g'(3) = 5$. If g^{-1} is the inverse function of g , find $\frac{dg^{-1}}{dx}$ when $x = 3$.

18. Use logarithmic differentiation to find y' if $y = \sqrt{\frac{x^2 + 1}{x + 1}}$.

19. The position function of an object moving along a straight line is given by $s(t) = t^4 - 4t + 1$, where position is measured in feet and time in seconds. Find the velocity and acceleration functions for this object. Find the total distance traveled by the object during the first two seconds.