```
> with(student): with(vec_calc): vc_aliases;
Warning, new definition for D
Warning, new definition for norm
Warning, new definition for trace
Package:   vec_calc   Version 4.3
For all HELP, execute: ?vec_calc
To use aliases, execute:   vc_aliases;
```

I, *Point*, *MF*, *Cv*, *Ca*, *Cj*, *CT*, *CN*, *CB*, *Ck*, *Ct*, *CL*, *CaT*, *CaN*, *Cforget*, *d2r*, *r2d*, *p2r*, *r2p*, *c2r*, *r2c*, *s2r*, *r2s*, *s2c*, *c2s*, *Muint*, *muint*, *LPMD*, *Lis*, *lis*, *Liv*, *liv*, *Sis*, *sis*, *Siv*, *siv*

Solutions to Exam II Version A

(Corresponding Problems in Version B in parenthesis after each problem)

#1 **A** Dominating Terms (#4 version B)

```
> Limit((1-2*exp(2*x))/(2+3*exp(2*x)),x=infinity);
  value(");
```

$$\lim_{x \to \infty} \frac{1 - 2\,\mathbf{e}^{(2\,x)}}{2 + 3\,\mathbf{e}^{(2\,x)}}$$

$$\frac{-2}{3}$$

#2 **D** Multiply numerator and denominator by 3, twice (since squared) (#5)

```
> Limit(sin(3*t)^2/t^2,t=0);
  value(");
```

$$\lim_{t \to 0} \frac{\sin(3\,t)^2}{t^2}$$

$$9$$

#3 **C** Use Chain Rule on each term (#3)

```
> f:=x->tan(x)^2+tan(x^2);
  D(f)(x):
  subs(1+tan(x)^2=sec(x)^2,1+tan(x^2)^2=sec(x^2),");
```

$$f := x \to \tan(x)^2 + \tan(x^2)$$

$$2\tan(x)\sec(x)^2 + 2\sec(x^2)\,x$$

#4 **D** Use Quotient Rule and Chain Rule (#1)

```
> f:=x->sin(x)^2/cos(x);
  D(f)(x);
```

$$f := x \to \frac{\sin(x)^2}{\cos(x)}$$

$$2\sin(x) + \frac{\sin(x)^3}{\cos(x)^2}$$

#5 **D** Use Chain Rule (#2)

```
> F:=f@g;
  D(F)(x);
  subs(x=3,D(g)(3)=4,g(3)=6,D(f)(6)=7,");
```

$$F := f@g$$

$$\mathrm{D}(f)(\mathrm{g}(x))\,\mathrm{D}(g)(x)$$

#6 **D** Use implicit differentiation and substitute to get slope (#6)

```
> eq:=y^2=x^3*(2-x);
  subs(y=y(x),eq);
  diff(",x);
  solve(",diff(y(x),x));
  subs(y(x)=1,x=1,");
  tanline_eqn:=y-1="*(x-1):
  tanline_eqn:=isolate(",y);
```

$$eq := y^2 = x^3 (2 - x)$$

$$\mathrm{y}(x)^2 = x^3 (2 - x)$$

$$2\, \mathrm{y}(x) \left(\frac{\partial}{\partial x} \mathrm{y}(x) \right) = 3\, x^2 (2 - x) - x^3$$

$$-\frac{x^2 (-3 + 2\, x)}{\mathrm{y}(x)}$$

$$1$$

$$tanline\_eqn := y = x$$

#7 **A** Differentiate by components, then substitute and divide by magnitude (#7)

```
> r:=MF(t,[cos(2*t),sin(2*t)]);
  D(r);
  D(r)(Pi/8);
  unit_tangent:="/len(");
```

$$r := [t \to \cos(2\, t), t \to \sin(2\, t)]$$

$$[t \to -2 \sin(2\, t), t \to 2 \cos(2\, t)]$$

$$[-\sqrt{2}, \sqrt{2}]$$

$$unit\_tangent := \left[-\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2} \right]$$

#8 **D** Use dy/dx = (dy/dt) / (dx/dt) to get slope (#11)

```
> x:=t->t*ln(t); y:=t->t*exp(2*t);
  dy_dx:=D(y)(t)/D(x)(t);
  slope:=D(y)(1)/D(x)(1);
```

$$x := t \to t \ln(t)$$

$$y := t \to t\, \mathbf{e}^{(2\, t)}$$

$$dy\_dx := \frac{\mathbf{e}^{(2\, t)} + 2\, t\, \mathbf{e}^{(2\, t)}}{\ln(t) + 1}$$

$$slope := 3\, \mathbf{e}^2$$

#9 **B** Use product and Chain Rule (#12)

```
> f:=x->x*exp(-x^2);
  D(f)(x);
```

$$f := x \to x\, \mathbf{e}^{(-x^2)}$$

$$\mathbf{e}^{(-x^2)} - 2\, x^2\, \mathbf{e}^{(-x^2)}$$

#10 **A** Use implicit differentiation and Chain Rule (Don't forget product rule for the exponent!) (#13)

```
> y:='y': eq:=2*exp(x*y)=x+y;
  subs(y=y(x),eq);
  diff(",x);
  solve(",diff(y(x),x));
  subs(y(x)=y,x=0,y=2,"): simplify(");
```

$$eq := 2\,\mathbf{e}^{(x\,y)} = x + y$$

$$2\,\mathbf{e}^{(x\,\mathrm{y}(x))} = x + \mathrm{y}(x)$$

$$2\left(\mathrm{y}(x) + x\left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)\right)\mathbf{e}^{(x\,\mathrm{y}(x))} = 1 + \left(\frac{\partial}{\partial x}\mathrm{y}(x)\right)$$

$$-\frac{2\,\mathbf{e}^{(x\,\mathrm{y}(x))}\,\mathrm{y}(x) - 1}{2\,\mathbf{e}^{(x\,\mathrm{y}(x))}\,x - 1}$$

$$3$$

#11 **D** Use logarithm properties: Don't forget to check your solutions! (#8)

```
> eq:=ln(x+6)+ln(x-3)=ln(5)+ln(2);
  eqalt:=ln((x+6)*(x-3))=ln(10);
  solve(eq,x);
```

$$eq := \ln(x+6) + \ln(x-3) = \ln(5) + \ln(2)$$

$$eqalt := \ln((x-3)\,(x+6)) = \ln(10)$$

$$4$$

#12 **C** Use Chain Rule (#9)

```
> f:=x->ln(x+ln(x));
  D(f)(x);
  D(f)(exp(1));
  simplify(");
```

$$f := x \rightarrow \ln(x + \ln(x))$$

$$\frac{\frac{1}{x} + 1}{x + \ln(x)}$$

$$\frac{\frac{1}{\mathbf{e}} + 1}{\mathbf{e} + 1}$$

$$\mathbf{e}^{(-1)}$$

#13 **D** Derivatives of sin(x) repeat every 4 derivatives (#10)

```
> f:='f';
  eq:=(D@@103)(f)(x)=(D@@3)(f)(x);
  (D@@3)(sin)(x);
```

$$f := f$$

$$eq := (D^{(103)})(f)(x) = (D^{(3)})(f)(x)$$

$$-\cos(x)$$

#14 (#18)

```
> r:='r': eq:=V(t)=4/3*Pi*r(t)^3;
```

```
diff(eq,t);
solve(",diff(r(t),t));
radius_change:=subs(diff(V(t),t)=-1,r(t)=5,");
eq2:=S(t)=4*Pi*r(t)^2;
diff(eq2,t);
subs(diff(r(t),t)=radius_change,r(t)=5,");
```

$$eq := V(t) = \frac{4}{3}\pi\, r(t)^3$$

$$\frac{\partial}{\partial t} V(t) = 4\pi\, r(t)^2 \left(\frac{\partial}{\partial t} r(t)\right)$$

$$\frac{1}{4}\frac{\frac{\partial}{\partial t} V(t)}{\pi\, r(t)^2}$$

$$radius\_change := -\frac{1}{100}\frac{1}{\pi}$$

$$eq2 := S(t) = 4\pi\, r(t)^2$$

$$\frac{\partial}{\partial t} S(t) = 8\pi\, r(t)\left(\frac{\partial}{\partial t} r(t)\right)$$

$$\frac{\partial}{\partial t} S(t) = \frac{-2}{5}$$

#15 (#19)

```
> eq:=y^5+3*x^2*y^2+5*x^4=12;
  subs(y=y(x),eq);
  diff(",x);
  solve(",diff(y(x),x));
  subs(y(x)=y,");
```

$$eq := y^5 + 3x^2 y^2 + 5x^4 = 12$$

$$y(x)^5 + 3x^2\, y(x)^2 + 5x^4 = 12$$

$$5\, y(x)^4\left(\frac{\partial}{\partial x} y(x)\right) + 6x\, y(x)^2 + 6x^2\, y(x)\left(\frac{\partial}{\partial x} y(x)\right) + 20x^3 = 0$$

$$-2\frac{x\,(3\, y(x)^2 + 10x^2)}{y(x)\,(5\, y(x)^3 + 6x^2)}$$

$$-2\frac{x\,(3y^2 + 10x^2)}{y\,(5y^3 + 6x^2)}$$

#16 Use similar triangles to set up (#16)

```
> eq:=w(t)/2=12/d(t);
  diff(eq,t);
  subs(diff(d(t),t)=16/10,d(t)=8,");
  solve(",diff(w(t),t));
```

$$eq := \frac{1}{2} w(t) = \frac{12}{d(t)}$$

$$\frac{1}{2}\left(\frac{\partial}{\partial t}\mathrm{w}(t)\right)=-12\,\frac{\frac{\partial}{\partial t}\mathrm{d}(t)}{\mathrm{d}(t)^2}$$

$$\frac{1}{2}\left(\frac{\partial}{\partial t}\mathrm{w}(t)\right)=\frac{-3}{10}$$

$$\frac{-3}{5}$$

#17a Switch x and y and solve for y (#17a)

```
> eq:=y=(x-2)/(x+2);
  neweq:=x=(y-2)/(y+2);
  solve(neweq,y);
```

$$eq := y = \frac{x-2}{x+2}$$

$$neweq := x = \frac{y-2}{y+2}$$

$$-2\,\frac{1+x}{x-1}$$

#17b Let f=g inverse. Use f’(x) = 1/ g’(f(x))

```
> f:='f': g:='g':
  D(g)(x)=1/D(g)(f(x));
  subs(x=3,f(3)=2,D(g)(2)=4,");
```

$$\mathrm{D}(g)(x)=\frac{1}{\mathrm{D}(g)(\mathrm{f}(x))}$$

$$\mathrm{D}(g)(3)=\frac{1}{4}$$

#18 (#14)

```
> eq:=y(x)=sqrt((x^2+1)/(x+1));  #NOT equal to x+1!!!!!!
  expand(ln(lhs(eq))=ln(rhs(eq)));
  diff(",x);
  solve(",diff(y(x),x));
```

$$eq := \mathrm{y}(x)=\sqrt{\frac{x^2+1}{1+x}}$$

$$\ln(\mathrm{y}(x))=\frac{1}{2}\ln(x^2+1)-\frac{1}{2}\ln(1+x)$$

$$\frac{\frac{\partial}{\partial x}\mathrm{y}(x)}{\mathrm{y}(x)}=\frac{x}{x^2+1}-\frac{1}{2}\,\frac{1}{1+x}$$

$$\frac{1}{2}\,\frac{\mathrm{y}(x)\,(2\,x+x^2-1)}{x^2+x^3+1+x}$$

#19 total distance=distance forward + distance backward (#15)

```
> s:=t->t^4-4*t+1;
```

```
velocity:=D(s)(t); #NOTE object turns around at t=1!!!
accel:=(D@@2)(s)(t);
distance:=abs(s(1)-s(0))+abs(s(2)-s(1));
```

$$s := t \rightarrow t^4 - 4\,t + 1$$

$$velocity := 4\,t^3 - 4$$

$$accel := 12\,t^2$$

$$distance := 14$$

```
>
```