```
> with(student): with(vec_calc): vc_aliases;
Warning, new definition for D
Warning, new definition for norm
Warning, new definition for trace
Package:   vec_calc   Version 4.3
For all HELP, execute: ?vec_calc
To use aliases, execute:   vc_aliases;
```

I, Point, MF, Cv, Ca, Cj, CT, CN, CB, Ck, Ct, CL, CaT, CaN, Cforget, d2r, r2d, p2r, r2p, c2r, r2c, s2r, r2s, s2c, c2s, Muint, muint, LPMD, Lis, lis, Liv, liv, Sis, sis, Siv, siv

Exam III

#1 Chain Rule and Inverse Trig Rule **C**

```
> f:=x->arctan(x^2);
  D(f)(x);
```

$$f := x \rightarrow \arctan(x^2)$$

$$2\,\frac{x}{1+x^4}$$

#2 Use reference triangle **A**

```
> sin(arctan(3/4));
```

$$\frac{3}{5}$$

#3 Mean Value Theorem **D**

```
> f:='f': MVT:=2<(f(4)-f(1))/(4-1);
  subs(f(1)=10,f(4)=y,MVT):
  solve(",y);
```

$$MVT := 2 < \frac{1}{3}\,\mathrm{f}(4) - \frac{1}{3}\,\mathrm{f}(1)$$

$$\mathrm{RealRange}(\mathrm{Open}(16), \infty)$$

#4 L'Hospital's Rule **A**

```
> Limit((sin(x)-x)/x^3,x=0)=limit((sin(x)-x)/x^3,x=0);
```

$$\lim_{x \to 0} \frac{\sin(x)-x}{x^3} = \frac{-1}{6}$$

#5 expand first, then set f'=0 and do first derivative test **D**

```
> f:=x->x^2*(1-x);
  expand(f(x));
  crit:=solve(D(f)(x)=0,x);
  f_inc:=solve(D(f)(x)>0,x);
```

$$f := x \rightarrow x^2\,(1-x)$$

$$x^2 - x^3$$

$$crit := 0, \frac{2}{3}$$

$$f\_inc := \mathrm{RealRange}\left(\mathrm{Open}(0), \mathrm{Open}\left(\frac{2}{3}\right)\right)$$

#6 find critical values, endpoints, and plug into f **B**

```
> f:=x->x^3-12*x;
  crit:=solve(D(f)(x)=0,x);
  f_neg3:=f(-3);
```

```
f_neg2:=f(-2);
f_2:=f(2)*min;
f_5:=f(5)*max;
```

$$f := x \to x^3 - 12\,x$$

$$crit := 2, \text{-}2$$

$$f\_neg3 := 9$$

$$f\_neg2 := 16$$

$$f\_2 := -16\ min$$

$$f\_5 := 65\ max$$

#7 f has an inflection point when f' changes direction (i.e., has a max or min) **C**

#8 f has a local max when f' changes from positive to negative **A**

#9 Expand and add **D**

```
> Sum(2*i-1,i=3..6)=sum(2*i-1,i=3..6);
```

$$\sum_{i=3}^{6} (2\,i - 1) = 32$$

#10 Integrate and substitute x and y to find the constant **A**

```
> df:=x->12*x^2-24*x+1;
  f:=int(df(x),x)+C;
  solve(-2=subs(x=1,f),C);
  subs(C=",f);
```

$$df := x \to 12\,x^2 - 24\,x + 1$$

$$f := 4\,x^3 - 12\,x^2 + x + C$$

$$5$$

$$4\,x^3 - 12\,x^2 + x + 5$$

#11 Integrate and substitute 0 and -1, then subtract **A**

```
> f:=x->(x+1)^3;
  Int(f(x),x=-1..0)=int(f(x),x=-1..0);
```

$$f := x \to (x+1)^3$$

$$\int_{-1}^{0} (x+1)^3\,dx = \frac{1}{4}$$

#12 Use Fundamental Theorem of Calculus, part I and Chain Rule **C**

```
> F:=x->Int(s^2/(s^2+1),s=1..sqrt(x));
  D(F)(x);
```

$$F := x \to \int_{1}^{\sqrt{x}} \frac{s^2}{s^2+1}\,ds$$

$$\frac{1}{2}\,\frac{\sqrt{x}}{x+1}$$

#13 Simplify first and then use the Power Rule

```
> f:=x->(x^2+1)/sqrt(x);
  expand(f(x));
  int(",x)+C;
```

$$f := x \rightarrow \frac{1 + x^2}{\sqrt{x}}$$

$$\frac{1}{\sqrt{x}} + x^{3/2}$$

$$2\sqrt{x} + \frac{2}{5} x^{5/2} + C$$

#14

```
> f:=x->sin(x)+cos(x);
  crit:=solve(D(f)(x)=0,x);
  f_0:=f(0)*min;
  f_pi_over_4:=f(Pi/4)*max;
  f_pi_over_3:=f(Pi/3);
```

$$f := x \rightarrow \sin(x) + \cos(x)$$

$$crit := \frac{1}{4}\pi$$

$$f\_0 := min$$

$$f\_pi\_over\_4 := \sqrt{2}\ max$$

$$f\_pi\_over\_3 := \frac{1}{2}\sqrt{3} + \frac{1}{2}$$

#15 Use first derivative test and second derivative number line

```
> f:=x->2*x^2-x^4;
  crit:=solve(D(f)(x)=0,x);
  f_inc:=solve(D(f)(x)>0,x);
  f_dec:=solve(D(f)(x)<0,x);
  inflpts:=solve((D@@2)(f)(x)=0,x);
  conc_up:=solve((D@@2)(f)(x)>0,x);
  conc_down:=solve((D@@2)(f)(x)<0,x);
```

$$f := x \rightarrow 2x^2 - x^4$$

$$crit := 0, 1, -1$$

$$f\_inc := \text{RealRange}(-\infty, \text{Open}(-1)), \text{RealRange}(\text{Open}(0), \text{Open}(1))$$

$$f\_dec := \text{RealRange}(\text{Open}(-1), \text{Open}(0)), \text{RealRange}(\text{Open}(1), \infty)$$

$$inflpts := \frac{1}{3}\sqrt{3}, -\frac{1}{3}\sqrt{3}$$

$$conc\_up := \text{RealRange}\left(\text{Open}\left(-\frac{1}{3}\sqrt{3}\right), \text{Open}\left(\frac{1}{3}\sqrt{3}\right)\right)$$

$$conc\_down := \text{RealRange}\left(-\infty, \text{Open}\left(-\frac{1}{3}\sqrt{3}\right)\right), \text{RealRange}\left(\text{Open}\left(\frac{1}{3}\sqrt{3}\right), \infty\right)$$

#16 Exponential Decay

```
> a:=y=200*exp(-ln(2)/140*t);
  b:=subs(t=100,a);
  c:=solve(subs(y=10,a),t);
```

$$a := y = 200\,\mathbf{e}^{(-1/140\ \ln(2)\ t)}$$

$$b := y = 200\,\mathbf{e}^{(-5/7\ \ln(2))}$$

$$c := 140\,\frac{\ln(20)}{\ln(2)}$$

#17 Take the logarithm and use L'Hospital's Rule (Remember to apply the exponential function when you are done!!!)

```
> Limit((1-2*x)^(1/x),x=0,right)=limit((1-2*x)^(1/x),x=0,right);
```

$$\lim_{x \to 0+} (1-2\,x)^{\left(\frac{1}{x}\right)} = \mathbf{e}^{(-2)}$$

#18 put length, width, and height in terms of x, the amount cut from each corner

```
> f:=x->(3-2*x)*(3-2*x)*x;
  expand(f(x));
  crit:=solve(diff(",x)=0,x);
  f_0:=f(0);
  f_three_halves:=f(3/2);
  f_crit:=f(crit[1]);
```

$$f := x \to (3-2\,x)^2\,x$$

$$9\,x - 12\,x^2 + 4\,x^3$$

$$crit := \frac{1}{2}, \frac{3}{2}$$

$$f\_0 := 0$$

$$f\_three\_halves := 0$$

$$f\_crit := 2$$

#19 I used similar triangles to get the proportion (x-8)/8 = x/y (where x is the length of the entire base of the large triangle) and substituted into minimizing c^2 = x^2 + y^2.

```
> min_length:=x^2+y^2;
  restriction:=(x-8)/8=x/y;
  solve(restriction,y);
  subs(y=",min_length);
  crit:=solve(diff(",x)=0,x);
  #first derivative test shows x=16 min.
  len:=sqrt(subs(x=16,y=16,min_length));
```

$$min\_length := x^2 + y^2$$

$$restriction := \frac{1}{8}\,x - 1 = \frac{x}{y}$$

$$8\,\frac{x}{x-8}$$

$$x^2 + 64\,\frac{x^2}{(x-8)^2}$$

$$crit := 0, 16, 4 - 4\,I\,\sqrt{3}, 4 + 4\,I\,\sqrt{3}$$

$$len := 16\,\sqrt{2}$$

```
>
```