MATH 151, FALL SEMESTER 2002

COMMON EXAMINATION II - VERSION A

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. In Part 1 (Problems 1–12), mark the correct choice on your ScanTron form (882-ES or 882-E) using a No:2 pencil. *For your own record, mark your choices on the exam itself*. ScanTrons will be collected from all examinees after one hour, and will *not* be returned.
2. Calculators may *not* be used in Part 1. The use of calculators is permitted *only after the first hour has elapsed and all ScanTrons have been collected*.
3. In Part 2 (Problems 13–17), present your solutions in the space provided. You may use the back of any page for rough work, but all work to be graded must be shown in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

QN	PTS
1–12	
13	
14	
15	
16	
17	
TOTAL	

Part 1 – Multiple Choice (48 points)

Read each question carefully; each problem is worth **4 points**. *Calculators are* **not** *allowed on this part of the exam.*

1. Find the slope of the tangent to the curve $y = 2x + \sec x$ at the point $(0, 1)$.

(a) 0 (b) 1 (c) 2 (d) 3 (e) the slope is not a finite number

2. Differentiate the function $f(x) = \cos^2 x$ with respect to x.

(a) $\sin^2 x$ (b) $2\cos x$ (c) $2\cos x \sin x$ (d) $-2\cos x \sin x$ (e) $-2x\sin^2 x$

3. Given that $f(x) = e^{-x}$, compute $f'(0)$.

(a) 1 (b) 0 (c) -1 (d) e (e) e^{-1}

4. Evaluate $\lim_{\theta \to 0} \dfrac{\sin\theta}{\pi - \theta}$.

(a) -1 (b) 0 (c) 1 (d) $1/\pi$ (e) $-1/\pi$

5. What is $\lim_{x \to \infty} \cos\left(\dfrac{1}{x}\right)$?

(a) 0 (b) 1 (c) -1 (d) $+\infty$ (e) $-\infty$

6. If $y^3 - x^2 = 1 - 2y$, find $\dfrac{dy}{dx}$ using implicit differentiation.

(a) $\dfrac{2x}{3y^2 + 2}$ (b) $\dfrac{1 + 2x}{3y^2 + 2}$ (c) $\dfrac{2x - 3y^2}{2}$ (d) $\dfrac{1 + 2x - 3y^2}{2}$ (e) $\dfrac{2x - 2y}{3y^2}$

7. If the equation of motion of a particle is given by $s = 2t^3 - 9t^2$ (s in meters and t in seconds), find the value(s) of t at which the acceleration is 0.

(a) $t = 0, 9/2$ (b) $t = 3/2$ (c) $t = 0, 3$ (d) the acceleration is never zero

8. Consider the curve given by the parametric equations $x(t) = t^3$, $y(t) = \sin(\pi t)$. Find the slope of the tangent to the curve at the point $(1, 0)$.

(a) $-1/3$ (b) $1/3$ (c) $-\pi/3$ (d) $\pi/3$ (e) 0

9. Obtain the tangent-line (*i.e.,* linear) approximation to the function $f(x) = \sqrt{1 + x^2}$ at the point $x = 0$.

(a) $y = 1 + x$ (b) $y = 1 + \dfrac{x}{2}$ (c) $y = 1 + 2x$ (d) $y = x$ (e) $y = 1$

10. Which of the following functions is **not** one-to-one on the given domain D?

(a) $f(x) = x$, $D = (-\infty, \infty)$ (b) $f(x) = 2x - 1$, $D = [0, \infty)$ (c) $f(x) = x^2$, $D = [0, 1]$
(d) $f(x) = x^2$, $D = [-1, 1]$ (e) $f(x) = x^3$, $D = [-1, 1]$

11. Let O and P denote the points on the plane with co-ordinates $(0, 0)$ and $(1, 0)$, respectively. For want of anything better to do, an ant begins to move from the point O along the y-axis, at a speed of $1/2$ unit per second. How fast is the distance between the ant and the point P changing, when he is 3 units away from O?

(a) $3\sqrt{10}/20$ (b) $3\sqrt{10}/10$ (c) $3/20$ (d) $3/2$ (e) 3

12. If $f^{(15)}(0) = 2$ and $H(x) = f\left(\dfrac{x}{2}\right)$, find $H^{(15)}(0)$.

(a) $1/2$ (b) 2^{14} (c) 2^{-15} (d) 2^{15} (e) 2^{-14}

Part 2 (56 points, including a bonus of 4 points)

Calculators are allowed for this part of the exam. Refer to the front page for further instructions.

13. Let C denote the curve given by the equation

$$x^3 - x^2y^2 + 3y - 1 = 0.$$

(i) (`7 points`) Use implicit differentiation to find $\dfrac{dy}{dx}$. Show all your steps.

(ii) (`3 points`) Obtain an equation of the tangent line to the curve C at the point $(-1, 1)$.

14. Let C denote the curve given by the vector equation

$$\mathbf{r}(t) = \langle\, e^{t^2+t}, \sqrt{1-3t}\,\rangle, \qquad -\infty < t \leq 1/3\,.$$

(i) (`6 points`) Find $\mathbf{r}'(t)$, $-\infty < t < 1/3$. Show all your steps.

(ii) (`2 points`) Find a tangent vector to the curve C corresponding to $t = 0$.

(iii) (`4 points`) Obtain a vector equation of the tangent line to the curve C corresponding to $t = 0$.

15. A street light is at the top of a pole which is 20 feet tall. A woman 6 feet tall walks away from the pole – along a straight path – at a speed of 5 feet per second. How fast is her shadow lengthening?

(i) (`4 points`) Draw a figure incorporating the given information, and label the appropriate variables.

(ii) (`8 points`) Complete the problem; include the appropriate unit in your final answer.

16. Suppose f is a differentiable function. Differentiate each of the following functions with respect to x.

(i) (`6 points`) $S(x) = 2e^{f(x)} - f(e^x)$

(ii) (`6 points`) $U(x) = f(x \sin x)$

17. Suppose F and G are differentiable functions. The line $y = 1 + 2x$ is the tangent-line approximation to F at $x = 2$, whereas the line $y = 2 - 3x$ is the tangent-line approximation to G at $x = 2$.

(i) (`4 points`) Find $F(2)$, $F'(2)$, $G(2)$, and $G'(2)$.

(ii) (`6 points`) Find the tangent-line approximation to $\dfrac{F}{G}$ at $x = 2$.