

Name _____

Math 151
Common Exam # 1, Version A
Spring 1998

Part I.

Each problem is worth 4 points. Mark your answers on your SCANTRON form. Indicate on your SCANTRON form whether you have Version A or B of the exam. For your own record, also mark your answers on this exam. The exam will be returned to you; the SCANTRON will not.

1. Find a unit vector in the direction of $3\mathbf{a} + 2\mathbf{b}$, where $\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j}$.

- a) $\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j}$ b) $\frac{1}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}$ c) $\frac{1}{\sqrt{3}}\mathbf{i} - \frac{2}{\sqrt{3}}\mathbf{j}$ d) $\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j}$

2. A triangle has vertices at the points $P(0, 1)$, $Q(2, 1)$, and $R(5, 4)$. Find the cosine of the angle of the triangle at the point R .

- a) $\frac{24}{\sqrt{18}\sqrt{34}}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{14}{\sqrt{41}\sqrt{5}}$ d) $\frac{4}{\sqrt{41}}$

3. Find the vector projection of $3\mathbf{i} - \mathbf{j}$ onto $\mathbf{i} + 2\mathbf{j}$.

- a) $\frac{3}{10}\mathbf{i} - \frac{1}{10}\mathbf{j}$ b) $\frac{1}{10}\mathbf{i} + \frac{1}{5}\mathbf{j}$ c) $\frac{1}{5}\mathbf{i} + \frac{2}{5}\mathbf{j}$ d) $\frac{3}{5}\mathbf{i} - \frac{1}{5}\mathbf{j}$

4. For which values of a are the vectors $(a + 2)\mathbf{i} + 3\mathbf{j}$ and $a\mathbf{i} - (2 + a)\mathbf{j}$ orthogonal?

- a) 2 and 0 b) 3 and -2 c) 0 and 3 d) 2 and -3

5. What is the domain of $f(x) = \frac{1}{\sqrt{4 - x^2}}$?

- a) $-2 \leq x \leq 2$ b) $x > 2$ or $x < -2$ c) $-2 < x < 2$ d) $x \geq 2$ or $x \leq -2$

6. A moving object's position at time t is given by the vector function $\mathbf{r}(t) = (5t + 3)\mathbf{i} + (2t - 16t^2 + 3)\mathbf{j}$. Find the average velocity of the object from time $t = 1$ to time $t = 3$.

- a) $10\mathbf{i} - 124\mathbf{j}$ b) $5\mathbf{i} + 62\mathbf{j}$ c) -57 d) $5\mathbf{i} - 62\mathbf{j}$

7. $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x^2 + 3x + 2} =$

- a) $-\frac{1}{3}$ b) 0 c) -3 d) does not exist

8. $\lim_{x \rightarrow 4} \frac{|x - 4|}{x - 4} =$

- a) 1 b) 0 c) -1 d) does not exist

9. $\lim_{x \rightarrow -\infty} \frac{2x^2 - x + 1}{\sqrt{25x^4 - 12x}} =$

- a) $\frac{2}{5}$ b) 0 c) ∞ d) does not exist

10. If $f(x) = \frac{6}{\sqrt{x}} - \frac{6}{x} + 6x - 6$, $f'(x) =$

- a) $6\sqrt{x} - \frac{6}{x^2} + 6$ b) $\frac{3}{x\sqrt{x}} - \frac{6}{x^2} + 6$ c) $3\sqrt{x} - \frac{6}{x^2} + 6$ d) $-\frac{3}{x\sqrt{x}} + \frac{6}{x^2} + 6$

11. Find $f'(t)$ if $f(t) = (2t^3 + 1)(7 - 3t)$.

- a) $-18t^2$ b) $-24t^3 + 42t^2 - 3$ c) $-12t^3 + 42t^2 + 3$ d) $24t^3 - 42t^2 + 3$

12. If $h(x) = \frac{f(x)}{g(x)}$, $f(2) = 1$, $f'(2) = -1$, $g(2) = 3$, and $g'(2) = 5$, find $h'(2)$.

- a) $-\frac{8}{9}$ b) $\frac{2}{9}$ c) $\frac{8}{9}$ d) $-\frac{2}{9}$

13. On which interval must the function $f(x) = 2 + x^2 - x^3 = 0$ have a root? Use the Intermediate Value Theorem.

- a) $[-1, 0]$ b) $[0, 1]$ c) $[1, 2]$ d) $[2, 3]$

Part II

Each problem is worth 8 points. Show all work on the exam sheets. No partial credit will be given for just an answer.

14. a) Using the definition of the derivative and taking limits, find $f'(1)$ if $f(x) = \frac{1}{2x - 1}$.

b) Check your answer to a) using the differentiation formulas.

15. If $\mathbf{r}(t) = \frac{\sqrt{4+t}-2}{t} \mathbf{i} + (t^2 \cos(\frac{1}{t}) + (t^2 + 8)^{4/3}) \mathbf{j}$, find $\lim_{t \rightarrow 0} \mathbf{r}(t)$.

16. For which values of a and b is the function

$$f(x) = \begin{cases} ax + b, & \text{if } x \leq 1; \\ 2bx^2 - 2ax + 3, & \text{if } x > 1. \end{cases}$$

differentiable at $x = 1$?

17. For which values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent line?

18. a) Two lines are given by $L_1(t) = \langle 5 + 3t, 2 + t \rangle$ and $L_2(t) = \langle 6 - 5t, 3 - t \rangle$. Are these parallel? Are they perpendicular? If they are not parallel, find their point of intersection.

b) Find the Cartesian, parametric, and vector equations of the straight line passing through the points $(2, -1)$ and $(5, 3)$.

19. A penguin, weighting 25 lbs, slides down an icy slope into the water. If the slope has an inclination of 30° to the horizontal and is 12 feet long, find the work done on the penguin by gravity.