Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points***. Calculator are* **not** *allowed for this part of the exam.*

1. If $\boldsymbol{u} = \langle 3, 5\rangle$ and $\boldsymbol{v} = \langle -2, 1\rangle$, compute $\boldsymbol{u} \cdot (\boldsymbol{u} + 2\boldsymbol{v})$.

   A: -1
   B: 28
   C: 7
   D: -9
   E: 32

2. Consider the points $P(-1, 3), Q(1, 1), R(2, -1)$. Compute $\cos\phi$, where ϕ is the angle at vertex P of triangle PQR.

   A: 14
   B: $\dfrac{7}{5\sqrt{2}}$
   C: $\dfrac{7}{\sqrt{2}}$
   D: $\dfrac{7}{5}$
   E: $\dfrac{14}{5}$

3. Let $\boldsymbol{u} = \langle 1, 2\rangle, \boldsymbol{v} = \langle -1, 3\rangle$ and $\boldsymbol{a} = \langle -4, 7\rangle$. If $\boldsymbol{a} = x\boldsymbol{u} + y\boldsymbol{v}$ find x.

   A: 1
   B: 0
   C: -1
   D: 2
   E: 3

4. Assume $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{u} - \boldsymbol{v}$ are unit vectors. Find $\boldsymbol{u} \cdot \boldsymbol{v}$.

   A: 0
   B: -1
   C: $\dfrac{1}{2}$
   D: $-\dfrac{1}{2}$
   E: Cannot be determined from the given data.

5. Let $f(x) = x^2 + 3x + 1$. Compute $\dfrac{f(2+h) - f(2)}{h}$.

   A: 1
   B: 7
   C: $7 + h$
   D: $5 - 2h$

E: $\dfrac{7}{h}$

6. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.

A: $y = \dfrac{x}{4}$

B: $y = \dfrac{x}{2}$

C: $y = \dfrac{x}{2} + 1$

D: $y = \dfrac{x}{4} + 1$

E: $y = \dfrac{x}{8} + \dfrac{3}{2}$

7. Compute: $\lim_{x\to 2} \dfrac{x^2-4}{x^2-x-2}$.

A: $\dfrac{4}{3}$

B: $\dfrac{3}{4}$

C: 0

D: $-\dfrac{3}{4}$

E: The limit does not exist.

8. Find $\lim_{x\to 2^-} \dfrac{|x-2|}{x^2-4}$

A: The limit does not exist.

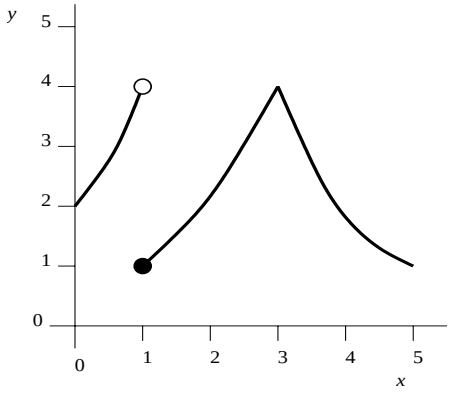
B: 1

C: -1

D: $-1/4$

E: 1/4

9. Below is the graph of $y = f(x)$. At which values of x does $f'(x)$ not exist?



A: 3 only

B: 1 and 4 only

C: 1 and 3 only

D: 1 only

E: $f(x)$ is differentiable everywhere

10. Compute $\lim_{x\to\infty} (\sqrt{x^2+3x+3} - \sqrt{x^2+4})$

A: ∞

B: 0

C: $-\infty$

D: 1

E: $\dfrac{3}{2}$

11. Consider the equation $x^3 - 3x + 1 = k$. For which of the following values of k does the intermediate value theorem guarantee that the equation has a solution in the interval (1,2)?

A: $k = -2$

B: $k = 2.5$

C: $k = \pi$

D: $k = 1000$

E: $k = -7.3$

12. Which of the following is an horizontal asymptote for the curve $y = \dfrac{2x+1}{3x^2+x-4}$?

A: $y = 0$

B: $y = \dfrac{2}{3}$

C: $y = 1$

D: $y = -\dfrac{4}{3}$

E: The curve has no horizontal asymptotes.

13. The functions $f(x)$ and $g(x)$ are differentiable at $x = 1$. We also know that $f(1) = 3$, $f'(1) = 2$, $g(1) = 3$, $g'(1) = -5$. Let $h = f(2f + g)$. Compute $h'(1)$

A: -2

B: 2

C: 15

D: 21

E: -21

PART 2 (52 points including a bonus of 4 points)

Refer to the front for instructions.

14. Consider the points $A(2,-1), B(3,5), C(-3,2)$.

a. (3pts) Find a vector equation for the line L joining A and B.

b. (3pts) Find a vector $\boldsymbol{n}$ perpendicular to L.

c. (3pts) Find $\text{comp}_{\boldsymbol{n}}(\overrightarrow{CA})$.

d. (3pts) Find the distance from C to the line L. Justify your answer.

15. a. (4pts) Give a precise definition of the derivative of a function $f(x)$ at the point $x = a$.

b. (4pts) Let $f(x)$ be a function whose graph passes through the point (4,3). The derivative of f is given by $f'(x) = \dfrac{1}{\sqrt{2x+1}}$. Compute $\lim\limits_{x \to 4} \dfrac{f(x) - 3}{x - 4}$. Justify your answer.

16. (8pts) Let $f(x) = \dfrac{x}{x+1}$. Use the definition of the derivative to compute $f'(2)$.
(**Note:** No credit will be given for using any other method, correct answer not withstanding.)

17. (8pts) Use the differentiation rules to compute $\dfrac{dy}{dx}$ if $y = \dfrac{2x-5}{x^2+3x-1}$

18. (8pts) Let $f(x) = \begin{cases} x^2 + ax + b & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ -ax^2 + 2bx + 6 & \text{if } x > 1 \end{cases}$

For which values of a and b is $f(x)$ continuous for all x?

19. The vector equation of a curve C is given by

$$\boldsymbol{r}(t) = \langle t-1, t^2-1 \rangle, \qquad -1 \leq t \leq 2.$$

a. (2pts) Write the parametric equations for the curve C.

b. (3pts) Eliminate the parameter t to find the Cartesian equation of C.

c. (3pts) Sketch a graph of C for the given interval for t. Use an arrow to indicate the direction of increasing t.