Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points**. *Calculator are* **not** *allowed for this part of the exam.*

1. Compute $\dfrac{d}{dx}\left(\dfrac{1}{\sqrt{e^x}}\right)$.

   A: $\dfrac{e^{-\sqrt{x}}}{2}$

   B: $\dfrac{1}{2\sqrt{e^x}}$

   C: $\dfrac{1}{2e^{\sqrt{x}}}$

   D: $\dfrac{2}{\sqrt{e^x}}$

   E: $-\dfrac{1}{2\sqrt{e^x}}$

2. Let $f(x)$ be a function differentiable for all x and let $g(x) = 3x^2 - 1$. Let $H(x) = f(g(x))$ be the composite function of f and g. The table below gives some values for $f(x)$ and $f'(x)$. Compute $H'(1)$.

   A: -5

   B: 30

   C: 12

   D: -6

   E: 6

x	0	1	2
$f(x)$	1	3	-1
$f'(x)$	-1	2	5

3. Find $\dfrac{d}{dx}(x\sin^2 x)$.

   A: $x\cos^2 x + \sin^2 x$

   B: $2\sin x\cos x$

   C: $1 + 2\sin x\cos x$

   D: $\sin^2 x + 2x\sin x\cos x$

   E: $x\cos^2 x$

4. Find $\lim_{x\to\infty} 3^{-x+1}$.

A: $\frac{1}{3}$

B: 3

C: 0

D: ∞

E: $-\infty$

5. Compute $\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}}$.

A: 1

B: $\frac{\pi}{2}$

C: ∞

D: 0

E: -1

6. If $\log_3 x + \log_3 x^2 = 6$, what is x?

A: 8

B: 3

C: 2

D: 1

E: 9

7. Which of the following is most likely the equation of the graph below?

A: $y = \log_{\frac{1}{2}} x$

B: $y = 2^x$

C: $y = \dfrac{1}{x}$

D: $y = 2^{-x}$

E: $y = \log_2 x$

8. If $L(x) = 3x + 2$ is the linear approximation of $f(x)$ around $x = -1$, which of the following statements is *false*.

A: The line $y = 3x + 2$ is tangent to the graph of $y = f(x)$ at $x = -1$.

B: In some small neighborhood of $x = -1, L(x)$ can be used as an approximation for $f(x)$.

C: If $L(x)$ is the linear approximation of another function $g(x)$ around $x = -1$ then $f(-1) = g(-1)$ and $f'(-1) = g'(-1)$.

D: $f(-1) = 2$

E: $f'(-1) = 3$

9. If $f(x) = 2e^{3x} - 1$ find $f^{-1}(3)$, where f^{-1} denotes the inverse function of $f(x)$.

A: $\dfrac{\ln 2}{3}$

B: $\dfrac{1}{2e^9 - 1}$

C: $\ln(2e^9 - 1)$

D: $-\dfrac{\ln 3}{2}$

E: $-\dfrac{\ln 2}{3}$

10. The linear approximation of $f(x) = \sqrt[3]{x}$ around $x = 64$ is used to estimate $\sqrt[3]{60}$. What is the value of that approximation?

A: $\dfrac{47}{12}$

B: $\dfrac{191}{48}$

C: 3.99

D: 62.5

E: 3.999

11. Assume that $y = e^{ax}$, where a is a constant, satisfies the relation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$, for all x. Find a.

A: There is no such value of a.

B: $a = 1$

C: $a = 0$

D: $a = 2$ only

E: $a = 2$ and $a = -2$

12. If Newton's method is used to solve the equation $x^3 + x = 3$ and the first guess is $x_1 = 1$, what is x_2?

A: $\dfrac{5}{4}$

B: $\dfrac{3}{2}$

C: $\dfrac{3}{4}$

D: $-\dfrac{1}{2}$

E: $\dfrac{1}{4}$

13. A point is moving on the curve $xy = 12$ $(x > 0)$. The x coordinate of the point is increasing at the rate of 4 units/min. At what rate is the y coordinate changing when $x = 4$?

A: 1

B: 4

C: -3

D: 3

E: -4

PART 2 (52 points including a bonus of 4 points)

Refer to the front for instructions.
You can use the following information if needed.

Special angle formulæ

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$

14. a) (6pts) Find the equation of the tangent line to the curve: $y^3 + x^2y = 3y - 1$ at the point $(1, 1)$

b) (4pts) Find y'' when $x = 1, y = 1$.

15. (6pts each) Find $\dfrac{dy}{dx}$ for each of the following.

a) $y = \dfrac{\cos x}{\sin x + \cos x}$, simplify your answer.

b) $y = \sec^3(\sqrt{x})$.

16. Let $f(x) = \dfrac{2x-1}{x+1}$ (note that $f(2) = 1$). Let $g(x)$ denote the inverse function of $f(x)$.

a) (4pts) Compute $g'(1)$ using the formula for the derivative of the inverse function.

b) (6pts) Compute $g(x)$ and $g'(x)$ and use your answers to verify your answer in part a).

17. The position vector of a particle moving in the plane is given by $\boldsymbol{r}(t) = \langle 3\sin(2t), 2\cos(2t)\rangle$.

a) (3pts) Find the velocity of the particle at $t = \dfrac{\pi}{6}$.

b) (1pt) Find the speed of the particle at $t = \dfrac{\pi}{6}$.

c) (3pts) Find the acceleration of the particle at $t = \dfrac{\pi}{6}$.

d) (3pts) Find the slope of the tangent line to the trajectory of the particle at $t = \dfrac{\pi}{6}$.

18. A police helicopter is flying along a straight highway at the constant height of .8 miles. The speed of the helicopter is 100 miles/hour. The radar gun determines that the distance of the helicopter to a car moving on the highway in the same direction ahead of the helicopter is 1 mile and decreasing at the rate of 18 miles/hour.

a) (2pts) Draw a diagram and label your variables on the diagram.

b) (8pts) Find the speed of the car.