Spring 2007
Math 151
Common Exam 2
Test Form A

PRINT: **Last Name** ______________________________ **First Name:** __________________

Signature: ____________________________________ **ID:** _________________________

Instructor's Name: ______________________________ **Section #** _____________

INSTRUCTIONS

In **Part 1** (Problems 1–12), mark the correct choice on your ScanTron form using a #2 pencil. *For your own records, also record your choices on your exam!* The ScanTrons will be collected after 1 hour; they will NOT be returned.

In **Part 2** (Problems 13–18), write all solutions in the space provided. CLEARLY INDICATE YOUR FINAL ANSWERS

No Calculators Permitted

Math 151 **Multiple Choice** **Part I**

(4 points each)

1. Find the limit

$$\lim_{x\to 0} \frac{2\cos(x)}{\sin(2x)(x+1)}$$

   (a) 0
   (b) $\frac{1}{2}$
   (c) 1
   (d) 2
   (e) does not exist

2. For $f(x) = \dfrac{2x-3}{7x+5}$, calculate $f^{-1}(x)$.

   (a) $f^{-1}(x)\dfrac{7x+5}{2x-3}$ (b) $f^{-1}(x) = \dfrac{5x+3}{-7x+2}$ (c) $f^{-1}(x) = \dfrac{2x-7}{3x+5}$

   (d) $f^{-1}(x) = \dfrac{2-3x}{7+5x}$ (e) $f^{-1}(x) = \dfrac{\frac{1}{2}x - \frac{1}{3}}{\frac{1}{7}x + \frac{1}{5}}$

3. Find the derivative of $f(x) = \dfrac{1+\cos x}{\sin x}$.

   (a) $f'(x) = \dfrac{-1-\cos x}{\sin^2 x}$ (b) $f'(x) = -\tan x$ (c) $f'(x) = \dfrac{1-\sin x}{\sin^2 x}$

   (d) $f'(x) = -\dfrac{1}{1+\cos x}$ (e) $f'(x) = \csc x + \cot x$

4. Calculate $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$.

(a) -1 (b) 0 (c) 1 (d) 2 (e) does not exist

5. Let f and g be functions with the following properties: $f(4) = 2$, $f'(4) = 2$, $g(1) = 4$ and $g'(1) = 2$. Compute $h'(1)$, where $h(x) = f(g(x))$.

(a) 2
(b) 4
(c) 6
(d) 12
(e) We do not have sufficient information.

6. Find the derivative of $f(x) = x^e + e^x$ at $x = 1$.

(a) 0 (b) 1 (c) e (d) 2 (e) $2e$

7. Find the equation of the line tangent to the curve $x^4 + y^2 = 25$ at the point (2,3).

(a) $y = \frac{3}{2}x$ (b) $y - 3 = -\frac{2x^3}{y}(x-2)$ (c) $y = -\frac{2}{3}x + \frac{13}{3}$

(d) $y - 3 = -\frac{16}{3}(x-2)$ (e) $y - 3 = \frac{y}{2x^3}(x-2)$

8. A ball follows a trajectory given in parametric form by $(x, y) = (\cos(t) + \sin(t), \cos^2 t + 1)$. What is the slope of the tangent line at the point (1,2)?

(a) 0

(b) $\frac{\cos^2(2)+1}{\cos(-1)+\sin(1)}$

(c) $-2\frac{\sin(2)\cos(2)}{\cos(1)-\sin(1)}$

(d) $-2\frac{\sin(1)\cos(1)}{\cos(2)-\sin(2)}$

(e) undefined

9. Let Q be a polynomial of degree 2 such that $Q(1) = 3$, $Q'(0) = -1$ and $Q''(0) = 2$. What is $Q(2)$?

(a) -1

(b) 1

(c) 3

(d) 5

(e) There is not enough information.

10. What is the differential of $f(x) = \sin(\cos x)$?

(a) $\sin(\cos(x))dx$

(b) $-\cos(\cos(x))\sin(x)$

(c) $-\sin(\cos(x))\sin(x)dx$

(d) $-\cos(\cos(x))\sin(x)dx$

(e) $(\sin(\cos(x)) + \sin(\sin(x)))dx$

11. Approximate the cosine of $\frac{1}{3}$ radians by using the quadratic approximation of $\cos x$ near $x = 0$.

(a) $\frac{1}{3}$ (b) $\frac{1}{18}$ (c) $\frac{17}{18}$ (d) $\frac{2}{3}$ (e) $\frac{8}{9}$

12. Suppose g is the inverse function of f. Assuming that $f(1) = 2$, $f'(2) = 3$, and $f'(1) = 3$ we can deduce that $g'(2)$ is

(a) 3

(b) 2

(c) 1

(d) 1/2

(e) 1/3

Show your work. No credit will be given to unsupported answers.

13. In the search for a solution of the equation $x^2 - 3 = 0$, consider the initial guess $x_1 = 2$.

    (a) Use Newton's method with $x_1 = 2$ as input to obtain a better approximation x_2 to a solution. (6 points)

    (b) Use Newton's method with the value of x_2 as input to obtain a still better approximation x_3 to a solution. (4 points).

14. A 13-foot ladder is leaning against the wall. If the foot of the ladder is sliding away from the wall at a constant rate of 7 feet per minute, how fast is the top of the ladder sliding down the wall when the foot of the ladder is 5 feet from the wall? (10 points)

15. Calculate both 1st and 2nd derivatives of $f(x) = \sqrt{3x - 2}$. (6 points)

16. In each case, find $\frac{dy}{dx}$ in terms of x and y.

(a) $e^{x+y} = x^2 + y$. (6 points)

(b) $3x^2 + y^3 + xy = 0$. (5 points)

17. Calculate the derivative of $f(x) = \tan(x^2 - x + 1)$. (5 points)

18. Let the position vector of a particle be given by $\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle$ at any given time t.

(a) Calculate the velocity as a vector function of t. (3 points)

(b) Calculate the acceleration as a vector function of t. (3 points)

(c) Verify that the velocity is perpendicular to the acceleration at all times. (4 points)