Part 1 – Multiple Choice (52 points)

Read each question carefully; each problem is worth **4 points**.
You can use the following information if needed.

Special angle formulæ

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$

1. Compute $\dfrac{d}{dx}(x\ln(4x))$.

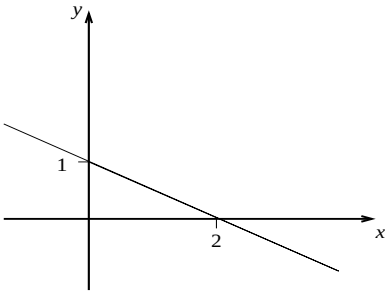
   A: $\ln(4x)$
   B: $1+\ln(4x)$
   C: $\frac{1}{4}+\ln(4x)$
   D: $\frac{1}{4x}$
   E: $\frac{4}{x}$

2. If $f'(x) = x(1-x^2)$ for $-3 \le x \le 2$, in which of the following intervals is $f(x)$ always increasing?

   A: $(-1, 0)$
   B: $(1, 2)$
   C: $(-3, 0)$
   D: $(-3, -2)$
   E: $(-1, 1)$

3. Below is the graph of the second derivative $f''(x)$ of the function $f(x)$. Which of the following must be true?

   A: $f(2) = 0$.
   B: $f(x)$ has a local maximum at $x = 2$.
   C: $f(x)$ has a local minimum at $x = 2$.
   D: $f'(2) = 0$.
   E: $f(x)$ has an inflection point at $x = 2$.



4. $\sin(\tan^{-1} x) =$

   A: $\frac{x}{\sqrt{1+x^2}}$
   B: $\frac{x}{\sqrt{1-x^2}}$
   C: $\frac{1}{\sqrt{1+x^2}}$
   D: $\frac{1}{\sqrt{1-x^2}}$
   E: $\sqrt{1+x^2}$

5. Compute $\lim_{x\to\frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{\frac{\pi}{3} - x}$.

   A: $-\sqrt{3}/2$
   B: $\sqrt{3}/2$
   C: $-1/2$
   D: $1/2$
   E: 1

6. Find the domain of $f(x) = 3\sin^{-1}(2x-5)$.

   A: $[-1, 1]$
   B: $[-\frac{\pi}{2}, \frac{\pi}{2}]$
   C: $(-\infty, \infty)$
   D: $[0, 1]$
   E: $[2, 3]$

7. The number $N(t)$ of bacteria in a culture triples every 10 hours. $N(t)$ satisfies the differential equation: $\frac{dN}{dt} = kN$. Compute k.

   A: $\frac{\ln 10}{3}$
   B: $\frac{\ln 3}{10}$
   C: $10 \ln 3$
   D: $\frac{3}{10}$
   E: $\frac{\ln 3}{\ln 10}$

8. Find the absolute maximum value and the absolute minimum values of $f(x) = x^3 - 2x^2 + x$ in $[-1, 1]$.

A: $\max = 2, \min = 0$

B: $\max = \frac{4}{27}, \min = -4$

C: $\max = 8, \min = -2$

D: $\max = 2, \min = -4$

E: $\max = \frac{15}{27}, \min = -4$

9. Which of the following is true for the function $f(x) = 3x^5 - 5x^3$ at the point $x = 0$?

A: The function has a discontinuity.

B: The function has a local maximum.

C: The function has a local minimum.

D: The function is continuous but it is not differentiable.

E: The function has an inflection point.

10. $\sum_{i=1}^{4} i(i-1) =$

A: 20

B: 12

C: 10

D: 24

E: 18

11. Compute $\int_0^3 |x - 2| \, dx$. (Hint: use areas.)

A: $\frac{7}{2}$

B: $\frac{13}{4}$

C: $\frac{9}{2}$

D: $\frac{5}{2}$

E: $\frac{17}{4}$

12. An antiderivative of xe^x is:

A: $\frac{x^2}{2}e^x$

B: $xe^x + e^x$

C: $xe^x - e^x$

D: x^2e^x

E: $\frac{1}{2}e^{x^2}$

13. Which of the definite integrals below is equal to $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} e^{1+\frac{i}{n}}$

A: $\int_0^1 e^x \, dx$

B: $\int_0^1 e^{\frac{x}{2}} \, dx$

C: $\int_1^2 e^x \, dx$

D: $\int_1^2 e^{1+x} \, dx$

E: $\int_0^2 e^{1+2x} \, dx$

PART 2 (52 points)

Refer to the front for instructions.

14. a) (6pts) Compute $\lim_{x \to +\infty} x^{\frac{1}{x}}$

b) (6pts) Compute $\lim_{x \to 0^+} x^{\frac{1}{x}}$

15. (8pts) Use logarithmic differentiation to compute the slope of the tangent line to the curve $y = \dfrac{(x-2)^3(3x-1)^{\frac{1}{3}}}{2\sqrt{x+1}}$ at $x = 3$.

16. The atmospheric pressure is often modeled by assuming that the rate of change of the pressure p with respect to the altitude x (height above sea level) is given by $\frac{dp}{dx} = kp$, where k is a constant. The atmospheric pressure at sea level is 1000 millibars and the pressure at 10 km is 250 millibars.

a) (4pts) Express the pressure p in terms of k and x.

b) (4pts) Compute k. (You can give your answer in terms of logarithms.)

c) (4pts) At what altitude will the pressure be 500 millibars? (You can give your answer in terms of logarithms.)

17. (10pts) A rectangular box with open top has height h, length l and width w. The length of the box is twice its width and the volume of the box is 9 ft^3. The material for the base costs \$10 per ft^2 and the material for the sides costs \$5 per ft^2. Find the dimensions of the box that will minimize the cost of the material. What is the minimal cost? Show that your answer gives a minimum.

18. A car is traveling at 125 ft/sec. The driver applies the brake giving the car an acceleration of $-10t$ ft/sec^2, where t is the time elapsed since the brake is applied.

a) (3pts) Find an expression for the velocity of the car at time t.

b) (2pts) How long will it take the car to come to a stop?

c) (5pts) What is the distance traveled between the time the brake is applied to the time the car comes to a stop?