

**MATH 152, SPRING SEMESTER 2005
COMMON EXAMINATION II - VERSION B**

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. Calculators may not be used on this exam. The ScanTrons will be collected after 90 minutes.
2. In Part 1 (Problems 1–10), mark the correct choice on your ScanTron form using a No.2 pencil. For your own record, mark your choices on the exam itself, as the ScanTrons will not be returned.
3. In Part 2 (Problems 11–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Possibly useful formulas:

$$|E_M| \leq \frac{K(b-a)^3}{24n^2}, \text{ where } K = \max|f''(x)| \text{ for } a \leq x \leq b$$

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}, \text{ where } K = \max|f''(x)| \text{ for } a \leq x \leq b$$

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}, \text{ where } K = \max|f^{(4)}(x)| \text{ for } a \leq x \leq b$$

<u>QN</u>	<u>PTS</u>
1–10	_____
11	_____
12	_____
13	_____
14	_____
15	_____
TOTAL	

Part 1 – Multiple Choice (50 points)

Read each question carefully; each problem is worth 5 points.

1. Find the x -coordinate of the center of mass of the system of objects that have a mass of 3, 5, and 7 at the points $(1, 2)$, $(-2, 5)$ and $(3, 1)$, respectively.

(a) $\frac{38}{15}$

(b) 0

(c) $\frac{14}{15}$

(d) $\frac{15}{14}$

(e) 14

2. If the Trapezoid Rule with $n = 4$ is used to approximate $\int_2^5 \ln x \, dx$, then the Trapezoid error formula says the error in the approximation is bounded by

(a) $|E_T| < \frac{9}{64}$

(b) $|E_T| < \frac{1}{256}$

(c) $|E_T| < \frac{9}{256}$

(d) $|E_T| < \frac{1}{64}$

(e) $|E_T| < \frac{1}{1024}$

3. The improper integral $\int_1^{\infty} \frac{1}{x^2 + 1} \, dx$

(a) Converges to 1.

(b) Converges to 0.

(c) Converges to $\frac{\pi}{2}$.

(d) Converges to $\frac{\pi}{4}$.

(e) Diverges.

4. Use The Trapezoid Rule rule with $n = 4$ to approximate $\int_1^2 \ln x \, dx$.
- (a) $\frac{1}{8}(2 \ln \frac{5}{4} + 2 \ln \frac{3}{2} + 2 \ln \frac{7}{4} + \ln 2)$
- (b) $\frac{1}{8}(\ln \frac{5}{4} + 2 \ln \frac{3}{2} + 2 \ln \frac{7}{4} + \ln 2)$
- (c) $\frac{1}{12}(4 \ln \frac{5}{4} + 2 \ln \frac{3}{2} + 4 \ln \frac{7}{4} + \ln 2)$
- (d) $\frac{1}{4}(\ln \frac{5}{4} + 2 \ln \frac{3}{2} + 2 \ln \frac{7}{4} + \ln 2)$
- (e) $\frac{1}{8}(2 \ln \frac{5}{4} + 2 \ln \frac{3}{2} + 2 \ln \frac{7}{4} + 2 \ln 2)$
5. If a curve passes thru the point $(0, \frac{1}{2})$ and has the property that the slope of the curve at each point (x, y) is x^3y^2 , find the equation of the curve.
- (a) $\frac{4}{8 - x^2}$
- (b) $\frac{4}{8 - x^4}$
- (c) $\frac{4}{x^4 - 8}$
- (d) $\frac{x^2}{4} + \frac{1}{2}$
- (e) $\frac{1}{2} - \frac{x^4}{4}$
- 6 Consider the integral $\int_0^{\infty} \frac{1}{\sqrt{x} + e^{10x}} \, dx$. Which of the following is true?
- (a) There is insufficient information to make a conclusion.
- (b) The integral diverges to infinity.
- (c) The integral converges to 0.
- (d) The integral converges to a value $L > \frac{1}{10}$
- (e) The integral converges to a value $L \leq \frac{1}{10}$

7. An integrating factor for the differential equation $(x^2 + 1)\frac{dy}{dx} + 4xy = x^3$ is:

(a) $I(x) = 4x \arctan x$

(b) $I(x) = 2x^2 + 2$

(c) $I(x) = (x^2 + 1)^2$

(d) $I(x) = e^{2x^2}$

(e) $I(x) = \ln(x^2 + 1)$

8. Which of the following integrals gives the surface area obtained by rotating the curve $y = \ln(4x)$, $1 \leq x \leq 3$ about the y -axis?

(a) $\int_1^3 2\pi x \sqrt{1 + \frac{1}{16x^2}} dx$

(b) $\int_1^3 2\pi x \sqrt{1 + \ln^2 4x} dx$

(c) $\int_1^3 2\pi \ln(4x) \sqrt{1 + \frac{1}{x^2}} dx$

(d) $\int_1^3 2\pi \ln(4x) \sqrt{1 + \frac{1}{16x^2}} dx$

(e) $\int_1^3 2\pi x \sqrt{1 + \frac{1}{x^2}} dx$

9. The length of the curve $x = 3t + 1$, $y = 4 - t$, $1 \leq t \leq 3$ is:

(a) $2\sqrt{10}$

(b) $10\sqrt{3}$

(c) $\frac{1}{15}(31\sqrt{31} - 11\sqrt{11})$

(d) $\frac{2}{3}(31\sqrt{31} - 11\sqrt{11})$

(e) $4\sqrt{10}$

10. Find the length of the curve $y = \frac{2}{3}(x - 1)^{\frac{3}{2}}$, $1 \leq x \leq 4$.

(a) $2\sqrt{3}$

(b) 9

(c) $\frac{12}{5}\sqrt{3}$

(d) $\frac{14}{3}$

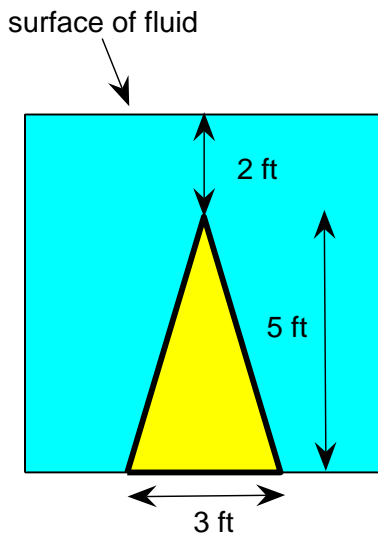
(e) $\sqrt{3}$

Part 2 (56 points)

The use of a calculator is *NOT* permitted for this part of the exam. All work must be shown in order to receive credit. Refer to the front page for further instructions.

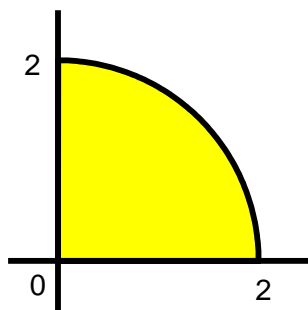
11. (12 points) A triangular shaped plate is submerged vertically in a fluid with weight density $\delta = \rho g = 60$ pounds per cubic feet as shown. Find the force of the fluid on one side of the plate.

Triangular plate submerged in fluid



12. (10 points) Find the surface area obtained by rotating the curve $x = y^2 + 1$, $1 \leq y \leq 2$ about the x -axis.

13. (10 points) Find the x -coordinate of the centroid of the quarter-circular region depicted.
Quarter-circular region



14. (14 points) Solve the differential equations. Solve explicitly for y .

a.) $\frac{dy}{dt} = 1 + t - yt - y$

b.) $\frac{dy}{dx} - 2y = x, y(0) = 1.$

- 15.** (10 points) A tank contains 1000 liters of pure water. Salt water that contains 0.5 kg of salt per liter of water enters the tank at a rate of 10 liters per minute. The solution is kept mixed and exits the tank at the same rate. How much salt is in the tank after 10 minutes?