

**MATH 152, SPRING SEMESTER 2005
COMMON EXAMINATION III - VERSION B**

Name (print): _____

Signature: _____

Instructor's name: _____

Section No: _____

Seat No: _____

INSTRUCTIONS

1. Calculators may not be used on this exam. The ScanTrons will be collected after 90 minutes.
2. In Part 1 (Problems 1–10), mark the correct choice on your ScanTron form using a No.2 pencil. For your own record, mark your choices on the exam itself, as the ScanTrons will not be returned.
3. In Part 2 (Problems 11–15), present your solutions in the space provided. **Show all your work** neatly and concisely, and **indicate your final answer clearly**. You will be graded, not merely on the final answer, but also on the quality and correctness of the work leading up to it.
4. Be sure to **write your name, section number, and version letter of the exam on the ScanTron form**.

Taylor's Inequality:

$$|R_n(x)| \leq \frac{M}{(n+1)!} |x-a|^{n+1} \text{ where } |f^{(n+1)}(x)| \leq M \text{ for } x \text{ in an interval containing } a.$$

QN	PTS
<u>1–10</u>	_____
<u>11</u>	_____
<u>12</u>	_____
<u>13</u>	_____
<u>14</u>	_____
<u>15</u>	_____
TOTAL	_____

Part 1 – Multiple Choice (50 points)

Read each question carefully; each problem is worth 5 points.

1. The sequence $a_n = \frac{(-10)^n}{n!}$
- (a) Diverges by oscillation.
 - (b) Converges to 0.
 - (c) Converges to -1 .
 - (d) Converges to 1.
 - (e) Converges to -10 .
2. Suppose it is known that the series $\sum_{n=1}^{\infty} c_n x^n$ converges at $x = 3$ and diverges at $x = 5$. Which of the following statements *must* be true?
- I. The series converges at $x = 4$.
 - II. The series diverges at $x = 6$.
 - III. The series converges at $x = 2$.
- (a) III only.
 - (b) I and III only.
 - (c) II and III only.
 - (d) I, II, and III.
 - (e) Cannot be determined without knowing c_n .
3. The series $\sum_{n=1}^{\infty} \frac{3(2)^{2n}}{5^{n+1}}$
- (a) Converges to $\frac{12}{5}$.
 - (b) Converges to 3.
 - (c) Converges to 1.
 - (d) Converges to $\frac{2}{5}$.
 - (e) Diverges.

4. Suppose $s_n = \frac{4n-5}{2+n}$ is the sequence of partial sums for the series $\sum_{n=1}^{\infty} a_n$. Then the sum of the series $\sum_{n=1}^{\infty} a_n$ is
- (a) 0
- (b) $\frac{5}{4}$
- (c) 2
- (d) 4
- (e) Divergent by the Test for Divergence.
5. Find $T_3(x)$, the third degree Taylor Polynomial, for $f(x) = \cos x$ at $x = \frac{\pi}{3}$.
- (a) $\frac{\sqrt{3}}{2} + \frac{1}{2}(x - \frac{\pi}{3}) + \frac{\sqrt{3}}{4}(x - \frac{\pi}{3})^2 + \frac{1}{12}(x - \frac{\pi}{3})^3$
- (b) $\frac{1}{2} + \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) + \frac{1}{4}(x - \frac{\pi}{3})^2 + \frac{\sqrt{3}}{12}(x - \frac{\pi}{3})^3$
- (c) $\frac{\sqrt{3}}{2} - \frac{1}{2}(x - \frac{\pi}{3}) - \frac{\sqrt{3}}{4}(x - \frac{\pi}{3})^2 + \frac{1}{12}(x - \frac{\pi}{3})^3$
- (d) $\frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) - \frac{1}{2}(x - \frac{\pi}{3})^2 + \frac{\sqrt{3}}{2}(x - \frac{\pi}{3})^3$
- (e) $\frac{1}{2} - \frac{\sqrt{3}}{2}(x - \frac{\pi}{3}) - \frac{1}{4}(x - \frac{\pi}{3})^2 + \frac{\sqrt{3}}{12}(x - \frac{\pi}{3})^3$
6. Which of the following is true about the series $\sum_{n=1}^{\infty} (-1)^n e^{1/n}$?
- (a) The series diverges by the Test for Divergence.
- (b) The series converges by the Alternating Series Test.
- (c) The series is a convergent Geometric Series.
- (d) The series converges by the Ratio Test.
- (e) The series diverges by the Ratio Test.

7. If we use a second degree Taylor Polynomial to approximate $f(x) = \frac{1}{x}$ at $x = 4$, use Taylor's Inequality to estimate the accuracy of the approximation $T_2(x) \approx f(x)$ for $2 \leq x \leq 6$.

(a) $|R_2(x)| \leq \frac{1}{27}$

(b) $|R_2(x)| \leq \frac{1}{18}$

(c) $|R_2(x)| \leq 1$

(d) $|R_2(x)| \leq \frac{1}{2}$

(e) $|R_2(x)| \leq 2$

8. Which of the following is a Maclaurin Series for $f(x) = x \cos(x^3)$?

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{9n+1}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+4}}{(2n+1)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+1}}{(2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{6n+3}}{(2n+1)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{3n+1}}{(2n)!}$

9. If we find the Taylor Series for $f(x) = \ln x$ at $x = 3$, what is the coefficient of $(x - 3)^3$?

(a) $\frac{1}{81}$

(b) $\frac{2}{27}$

(c) $-\frac{1}{81}$

(d) $-\frac{2}{27}$

(e) $\frac{2}{81}$

10. The sum of the series $\sum_{n=2}^{\infty} \left(\cos \frac{1}{n} - \cos \frac{1}{n+1} \right)$ is:

(a) 0

(b) $\cos \left(\frac{1}{2} \right) - 1$

(c) $\cos \left(\frac{1}{3} \right)$

(d) $\cos \left(\frac{1}{2} \right)$

(e) The series diverges by oscillation.

Part 2 (56 points)

The use of a calculator is NOT permitted for this part of the exam. All work must be shown in order to receive credit. Refer to the front page for further instructions.

- 11. (12 points)** Find the radius and interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-2)^n (x+1)^n}{n}$.

- 12.** (10 points) Define Absolute Convergence. Does the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^4}$ converge absolutely? Justify your answer.

13. (12 points total) Consider $\int_0^{0.1} e^{-x^2} dx$.

a.) (5 pts) Express $\int_0^{0.1} e^{-x^2} dx$ as an infinite series.

b.) (3 pts) Use the first three terms of the series to obtain an approximation to $\int_0^{0.1} e^{-x^2} dx$.
You need not simplify.

c.) (4 pts) Find an upper bound on the error in this approximation. Justify your answer.

14. (10 points) Determine whether the following series converge or diverge. Fully support your answer.

a.)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/4}}$$

b.)
$$\sum_{n=2}^{\infty} \frac{n^2}{n^4 - n}$$

15. (12 points) Write $f(x) = \ln(1 + 8x)$ as a power series. What is the associated radius of convergence?