

SOLUTIONS: Executive Summary [method]

1. (d) The cosine of the angle θ between the vectors $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is $\cos \theta = \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \|\mathbf{w}\|} = \frac{1}{14}$.
2. (a) The vector projection is $\text{proj}_{\mathbf{a}} \mathbf{b} = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\|^2} \right) \mathbf{a} = \frac{160}{129} \mathbf{i} - \frac{320}{129} \mathbf{j} - \frac{280}{129} \mathbf{k}$.
3. (c) The series $\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ converges by the Integral Test.
4. (e) The sum of the geometric series $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{5^n} = \sum_{n=1}^{\infty} \left(\frac{1}{5} \right) \left(-\frac{3}{5} \right)^{n-1}$ is $\frac{1/5}{1 - (-3/5)} = \frac{1}{8}$.
5. (b) The sequence $a_n = (-1)^n \frac{n}{n+2}$, $n \geq 1$, is bounded, yet nonmonotonic.
6. (a) The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ is convergent, but not absolutely convergent by the Alternating Series Test and Integral Test.
7. (d) The power series $\sum_{n=0}^{\infty} \frac{n^2}{2^n} (x-3)^n$ converges for $1 < x < 5$ via the Root Test (or Ratio Test) and Test for Divergence.
8. (d) The second-degree Taylor polynomial for $f(x) = \sqrt{x+3}$ about $a = 1$ is $T_2(x) = 2 + \frac{1}{4}(x-1) - \frac{1}{64}(x-1)^2$.
9. The function $f(x) = \frac{x}{1+x^3}$ has a Taylor series expansion about $a = 1$ given by $f(x) = \frac{1}{2} - \frac{1}{4}(x-1) - \frac{3}{8}(x-1)^2 + \dots$, from which the relevant derivative and coefficient information may be ascertained.
10. (a) The Maclaurin series expansion for the sine of t is $\sin t = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} t^{2n+1}$, valid for $t \in \mathbb{R}$.
- (b) Via algebra and term-by-term antidifferentiation, the power series expansion for the sine integral $\text{Si}(x) = \int_0^x \frac{\sin t}{t} dt$ is $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!(2n+1)}$, valid for $x \in \mathbb{R}$.
11. Consider the series $\sum_{n=1}^{\infty} \frac{(-5)^{n+1}}{(2n+1)!}$.
- (a) It converges absolutely via the Ratio Test.
- (b) We have $s_2 = 3\frac{1}{8}$ or equivalent.
- (c) The Alternating Series Estimation Theorem yields an upper bound on the remainder in using s_2 to approximate the sum of the series, namely $125/1008$ or equivalent.
12. Via the Geometric Series Theorem, the power series expansion about $a = 0$ for $f(x) = \frac{1}{1+9x^2}$ is $\sum_{n=0}^{\infty} (-1)^n 9^n x^{2n}$.
- Its radius of convergence is $R = \frac{1}{3}$.