

Fall 2009 Math 152

Exam I Version A Solutions

1. **C** Let $u = 2 + x^4$. Then $du = 4x^3 dx$. Substituting into the integral yields $\frac{1}{4} \int u^{1/2} du = \frac{1}{6} u^{3/2} + C = \frac{(2 + x^4)^{3/2}}{6} + C$

2. **D** Let $u = 1 + e^x$. Then $du = e^x dx$. Substituting into the integral yields $\int u^{10} du = \frac{1}{11} u^{11} + C = \frac{(1 + e^x)^{11}}{11} + C$

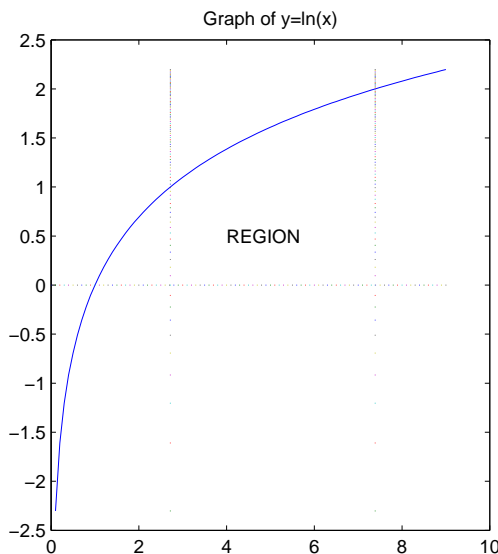
3. **A** Let $u = 2 + \tan \theta$. Then $du = \sec^2 \theta d\theta$. When $\theta = 0$, $u = 2$, and when $\theta = \frac{\pi}{4}$, $u = 3$. Substituting into the integral yields $\int_2^3 \frac{du}{u} = \ln |u| \Big|_2^3 = \ln 3 - \ln 2 = \ln \left(\frac{3}{2} \right)$.

4. **A** Applying the formula, $f_{avg} = \frac{1}{b-0} \int_0^b (2+6x) dx = \frac{1}{b} (2x+3x^2) \Big|_0^b = \frac{2b+3b^2}{b} = 2+3b = 3$. Solving this equation for b yields $b = \frac{1}{3}$.

5. **E** Integrate by parts, with $u = x$, $dv = e^{-2x}$. Then $du = dx$, $v = -\frac{1}{2}e^{-2x}$. $\int x e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \int -\frac{1}{2} e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C = -\frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$.

6. **D** Using an identity for $\sin^2 x$: $\int 2 \sin^2 \theta d\theta = \int 2 \cdot \frac{1}{2} (1 - \cos 2\theta) d\theta = \theta - \frac{1}{2} \sin 2\theta + C = \theta - \frac{\sin 2\theta}{2} + C$

7. **B** From the graph below, $A = \int_e^{e^2} \ln x dx$. Integrate by parts with $u = \ln x$, $dv = dx$. Then $du = \frac{dx}{x}$, $v = x$. $A = x \ln x \Big|_e^{e^2} - \int_e^{e^2} x \cdot \frac{dx}{x} = 2e^2 - e - (e^2 - e) = e^2$.



8. **E** Take a thin sheet of water with thickness dx at a distance x meters from the top of the tank. The weight of the sheet (density times volume) is $(10^3)(9.8)(2)(1)(dx) N$. The distance this sheet travels before leaving the tank is x meters. The total work required to pump half the water out is $\int_0^{1/2} (19.6)(10^3)x dx = (9.8)(10^3)x^2 \Big|_0^{1/2} = 2.45 \times 10^3 J$

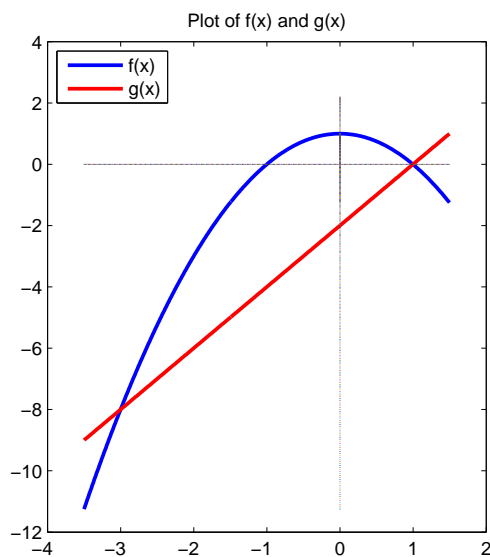
9. **D** Slice into cross-sections perpendicular to the y -axis with thickness dy . The volume of a slice is $\frac{1}{2}\pi \left(\frac{1}{2}x \right)^2 dy$. Since $y = \sqrt{x}$, $x = y^2$, so the total volume is given by $\int_0^1 \frac{\pi}{8} y^4 dy = \frac{\pi}{40} y^5 \Big|_0^1 = \frac{\pi}{40}$.

10. **C** Let $u = 2x - 3$. Then $du = 2 dx$. Substituting into the integral gives us $\int g(x) dx = \int f(2x - 3) dx = \frac{1}{2} \int f(u) du = \frac{F(u)}{2} + C = \frac{F(2x - 3)}{2} + C$.

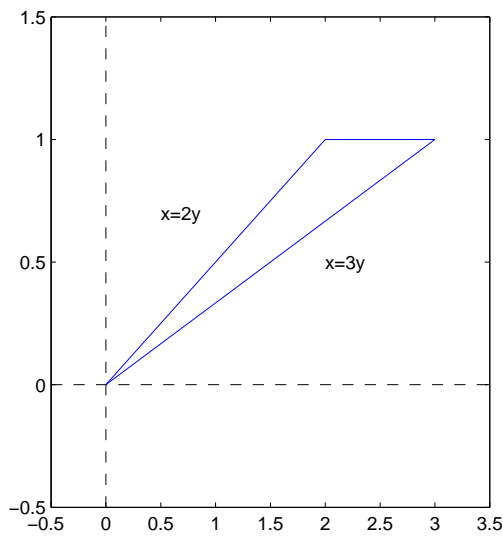
11. $\int \tan^3 x \sec^3 x dx = \int \tan^2 x \sec^2 x (\sec x \tan x dx) = \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x dx)$. Let $u = \sec x$. Then $du = \sec x \tan x dx$. Substituting into the integral yields $\int (u^2 - 1)u^2 du = \int (u^4 - u^2) du = \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$.

12. Let $u = \sqrt{x}$. Then $du = \frac{dx}{2\sqrt{x}}$. Substituting into the integral yields $2 \int \sin^3 u \, du = 2 \int \sin^2 u (\sin u \, du) = 2 \int (1 - \cos^2 u) (\sin u \, du)$. Let $w = \cos u$. Then $dw = -\sin u \, du$. Substituting into the integral yields $-2 \int (1 - w^2) \, dw = -2(w - \frac{w^3}{3}) + C = -2(\cos u - \frac{\cos^3 u}{3}) + C = -2(\cos \sqrt{x} - \frac{\cos^3 \sqrt{x}}{3}) + C$.

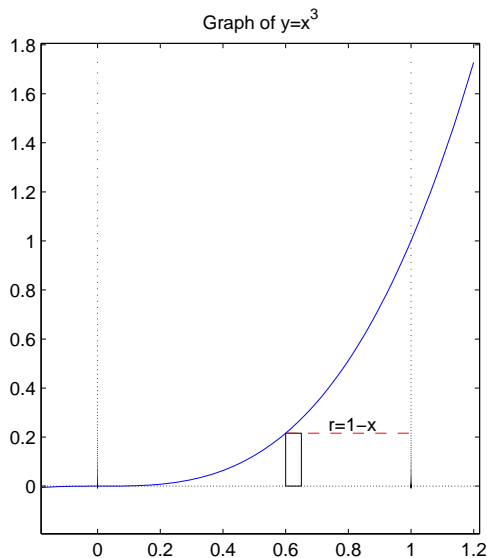
13. The plot is below. The curves intersect when $1 - x^2 = 2x - 2$. Solving for x yields $x^2 + 2x - 3 = 0$, $(x + 3)(x - 1) = 0$, or $x = -3, 1$. From the graph, the area is $\int_{-3}^1 ((1 - x^2) - (2x - 2)) \, dx = \int_{-3}^1 (-x^2 - 2x + 3) \, dx = -\frac{x^3}{3} - x^2 + 3x \Big|_{-3}^1 = \left(-\frac{1}{3} - 1 + 3\right) - (9 - 9 - 9) = \frac{32}{3}$



14. The graph of the region, with equations of the appropriate lines (as functions of y), is shown below. The volume obtained by rotating this region about the y -axis is $V = \pi \int_0^1 ((3y)^2 - (2y)^2) \, dy = \pi \int_0^1 5y^2 \, dy = \pi \frac{5}{3} y^3 \Big|_0^1 = \frac{5\pi}{3}$.



15. The graph of the region is shown below, along with an arbitrary strip which, when rotated about $x = 1$, produces a cylindrical shell. The volume of that shell is $2\pi(1-x)(x^3) \, dx$, so the volume of the solid is $V = \int_0^1 2\pi(x^3 - x^4) \, dx = 2\pi \left(\frac{x^4}{4} - \frac{x^5}{5}\right) \Big|_0^1 = 2\pi \left(\frac{1}{4} - \frac{1}{5}\right) = \frac{\pi}{10}$.



16. Integrate by parts, with $u = \sin x$, $dv = f''(x) \, dx$. Then $du = \cos x$, $v = f'(x)$. Then $\int_0^\pi f''(x) \sin x \, dx = f'(x) \sin x \Big|_0^\pi - \int_0^\pi f'(x) \cos x \, dx$. Integrate by parts again, with $u = \cos x$, $dv =$

$$\begin{aligned}
& f'(x) dx. \text{ Then } du = -\sin x dx, v = f(x). \\
& \text{The integral becomes } \int_0^\pi f'(x) \sin x dx - \\
& (f(x) \cos x)|_0^\pi - \int_0^\pi -f(x) \sin x dx. \\
& = f'(\pi) \sin \pi - f'(0) \sin 0 - (f(\pi) \cos \pi - \\
& f(0) \cos 0) - \int_0^\pi f(x) \sin x dx \\
& = 0 - 0 - ((-1)(-1) - (1)(1)) - 3 = -3.
\end{aligned}$$