Fall 2009 Math 152

Exam I Version B Solutions

1. **B** Let $u = 2 + x^4$. Then $du = 4x^3 dx$. Substituting into the integral yields $\frac{1}{4}\int u^{1/2}\,du = \frac{1}{6}u^{3/2} + C = \frac{(2+x^4)^{3/2}}{6} + C$

2. **C** Let $u = 1 + e^x$. Then $du = e^x\,dx$. Substituting into the integral yields $\int u^{10}\,du = \frac{1}{11}u^{11} + C = \frac{(1+e^x)^{11}}{11} + C$

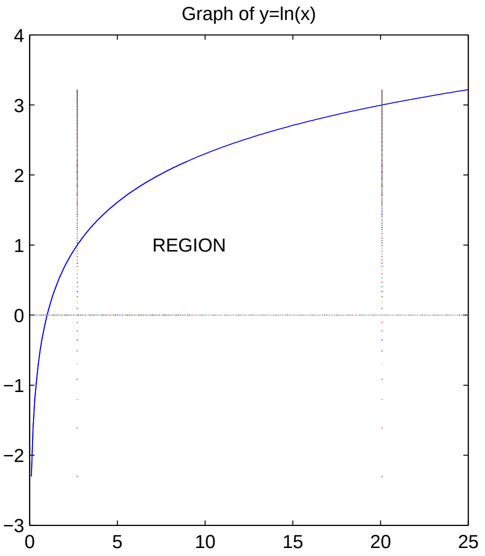
3. **E** Let $u = 3 + \tan\theta$. Then $du = \sec^2\theta\,d\theta$. When 3, and when $\theta = \frac{\pi}{4}$, $u = 4$. Substituting into the integral yields $\int_3^4 \frac{du}{u} = \ln|u|\big|_3^4 = \ln 4 - \ln 3 = \ln\left(\frac{4}{3}\right)$.

4. **B** Applying the formula, $f_{avg} = \frac{1}{b-0}\int_0^b (2+6x)\,dx = \frac{1}{b}\left(2x + 3x^2\big|_0^b\right.$. $= \frac{2b+3b^2}{b} = 2 + 3b = 3$. Solving this equation for b yields $b = \frac{1}{3}$.

5. **A** Integrate by parts, with $u = x$, $dv = e^{-3x}$. Then $du = dx$, $v = -\frac{1}{3}e^{-3x}$. $\int xe^{-3x}\,dx = -\frac{1}{3}xe^{-3x} - \int -\frac{1}{3}e^{-3x}\,dx$ $= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C = -\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9} + C$.

6. **E** Using an identity for $\cos^2 x$: $\int 2\cos^2\theta\,d\theta = \int 2\cdot\frac{1}{2}(1+\cos 2\theta)\,d\theta = \theta + \frac{1}{2}\sin 2\theta + C = \theta + \frac{\sin 2\theta}{2} + C$

7. **D** From the graph below, $A = \int_e^{e^3} \ln x\,dx$. Integrate by parts with $u = \ln x$, $dv = dx$. Then $du = \frac{dx}{x}$, $v = x$. $A = x\ln x\big|_e^{e^3} - \int_e^{e^3} x\cdot\frac{dx}{x} = 3e^3 - e - (e^3 - e) = 2e^3$.



8. **D** Take a thin sheet of water with thickness dx at a distance x meters from the top of the tank. The weight of the sheet (density times volume) is $(10^3)(9.8)(2)(1)(dx)\ N$. The distance this sheet travels before leaving the tank is x meters . The total work required to pump half the water out is $\int_0^{1/2}(19.6)(10^3)x\,dx = (9.8)(10^3)x^2\big|_0^{1/2} = 2.45\times 10^3 J$

9. **E** Slice into cross-sections perpendicular to the y-axis with thickness dy. The volume of a slice is $\frac{1}{2}\pi\left(\frac{1}{2}x\right)^2 dy$. Since $y = \sqrt{x}$, $x = y^2$, so the total volume is given by $\int_0^1 \frac{\pi}{8}y^4\,dy = \frac{\pi}{40}y^5\big|_0^1 = \frac{\pi}{40}$.

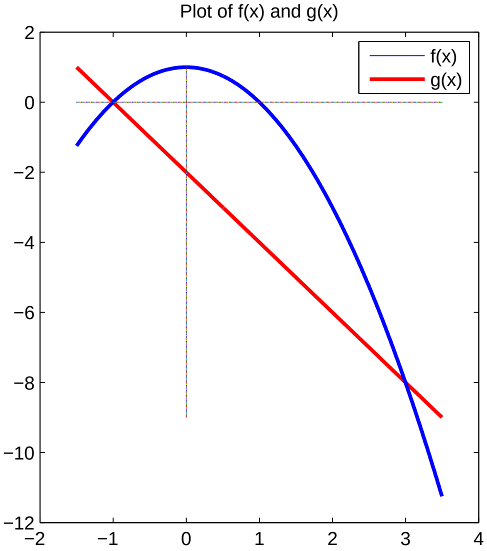
10. **D** Let $u = 3x - 2$. Then $du = 3\,dx$. Substituting into the integral gives us $\int g(x)\,dx = \int f(3x-2)\,dx = \frac{1}{3}\int f(u)\,du = \frac{F(u)}{3} + C = \frac{F(3x-2)}{3} + C$.

11. Let $\underline{u = \sqrt{x}.}$ Then $du = \frac{dx}{2\sqrt{x}}$. Substituting into the integral yields $2\int\cos^3 u\,du = 2\int\cos^2 u\,(\cos u\ \ du) = 2\int(1-\sin^2 u)(\cos u\ du)$. Let $w = \sin u$. Then $dw = \cos u\ du$. Substituting into the integral yields $2\int(1 - w^2)\,dw =$

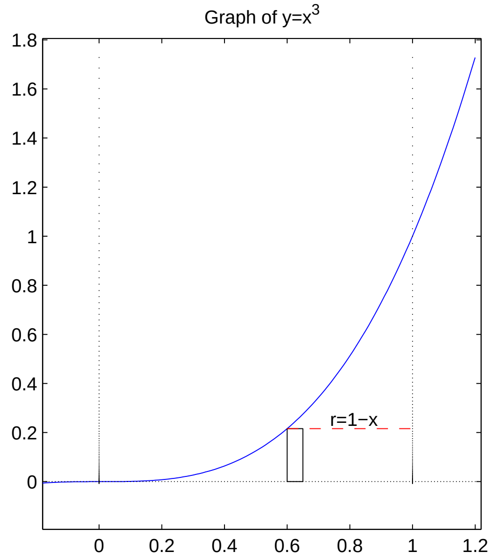
$2(w - \frac{w^3}{3}) + C = 2(\sin u - \frac{\sin^3 u}{3}) + C = 2(\sin \sqrt{x} - \frac{\sin^3 \sqrt{x}}{3}) + C.$

12. $\int \sec^3 x \tan^3 x \, dx = \int \sec^2 x \tan^2 x \, (\sec x \tan x \, dx) = \int (\sec^2 x - 1) \sec^2 x (\sec x \tan x \, dx)$. Let $u = \sec x$. Then $du = \sec x \tan x \, dx$. Substituting into the integral yields $\int (u^2 - 1) u^2 \, du = \int (u^4 - u^2) \, du = \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C.$

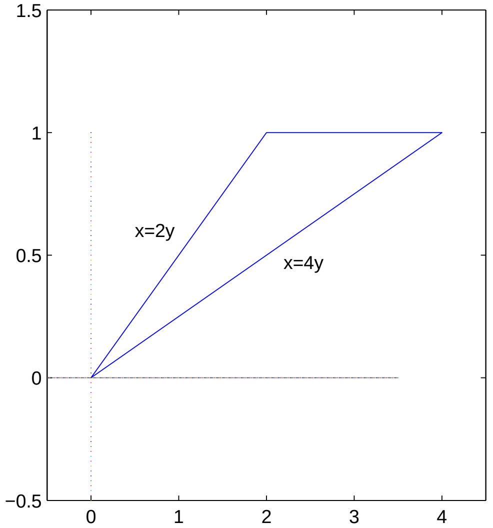
13. The plot is below. The curves intersect when $1 - x^2 = -2x - 2$. Solving for x yields $x^2 - 2x - 3 = 0$, $(x-3)(x+1) = 0$, or $x = 3, -1$. From the graph, the area is $\int_{-1}^{3} ((1 - x^2) - (-2x - 2)) \, dx = \int_{-1}^{3} (-x^2 + 2x + 3)$ $= -\frac{x^3}{3} + x^2 + 3x \Big|_{-1}^{3} = (-9 + 9 + 9) - (\frac{1}{3} + 1 - 3) = \frac{32}{3}$



14. The graph of the region is shown below, along with an arbitrary strip which, when rotated about $x = 1$, produces a cylindrical shell. The volume of that shell is $2\pi(1 - x)(x^3) \, dx$, so the volume of the solid is $V = \int_0^1 2\pi(x^3 - x^4) \, dx = 2\pi \left(\frac{x^4}{4} - \frac{x^5}{5} \Big|_0^1 \right.$. $= 2\pi \left(\frac{1}{4} - \frac{1}{5} \right) = \frac{\pi}{10}$



15. The graph of the region, with equations of the appropriate lines (as functions of y), is shown below. The volume obtained by rotating this region about the y-axis is $V = \pi \int_0^1 ((4y)^2 - (2y)^2) \, dy = \pi \int_0^1 12y^2 \, dy = \pi \; 4y^3 \big|_0^1 = 4\pi.$



16. Integrate by parts, with $u = \sin x$, $dv = f''(x) \, dx$. Then $du = \cos x$, $v = f'(x)$. Then $\int_0^{\pi} f''(x) \sin x \, dx = f'(x) \sin x|_0^{\pi} - \int_0^{\pi} f'(x) \cos x \, dx.$ Integrate by parts again, with $u = \cos x$, $dv =$

$f'(x)\,dx$. Then $du = -\sin x\,dx$, $v = f(x)$. The integral becomes $f'(x)\sin x|_0^\pi - (f(x)\cos x|_0^\pi - \int_0^\pi -f(x)\sin x\;dx)$.
$= f'(\pi)\sin\pi - f'(0)\sin 0 - (f(\pi)\cos\pi - f(0)\cos 0) - \displaystyle\int_0^\pi f(x)\sin x\;dx$
$= 0 - 0 - ((1)(-1) - (-1)(1)) - 4 = -4.$